2015-16

Lesson 1: Graphs of Piecewise Linear Functions

Graphs of Piecewise Linear Functions

When watching a video or reading a graphing story, the horizontal axis usually represents time, and the vertical axis represents a height or distance. Depending on the details of the story, different time intervals will be represented graphically with different line segments.

Create an Elevation-Versus-Time Graph for a Story

Elevation refers to a distance above or below a reference point, usually ground level where the elevation is 0 units of length.

Read the story below, and construct an elevation-versus-time graph that represents this situation.

Betty lives on the third floor. At time $t = 0$ seconds, she walks out her door. After 10 seconds she is at the third floor landing and goes downstairs. She reaches the second floor landing after 20 more seconds and realizes that she forget her phone. She turns around to go back upstairs at the same pace she went down the stairs. It takes her two minutes to grab her phone once she reaches the third floor landing. Then she quickly runs down all three flights of stairs and is on the ground floor 45 seconds later. Assume that the change in elevation for each flight of stairs is 12 feet.

If each flight of stairs is 12 feet and Betty lives on the third floor, then her highest elevation will be 36 feet. I will measure her elevation from her feet, not the top of her head.

1. Draw your own graph for this story. Use straight line segments to model Betty's elevation over different time intervals. Label your horizontal and vertical axes, and title your graph.

There are 5 time intervals: going to the stairs, going down to the second floor, going back up, getting the phone, and going down all three flights.

I will label the horizontal axis time measured in seconds and the vertical axis elevation measured in feet.

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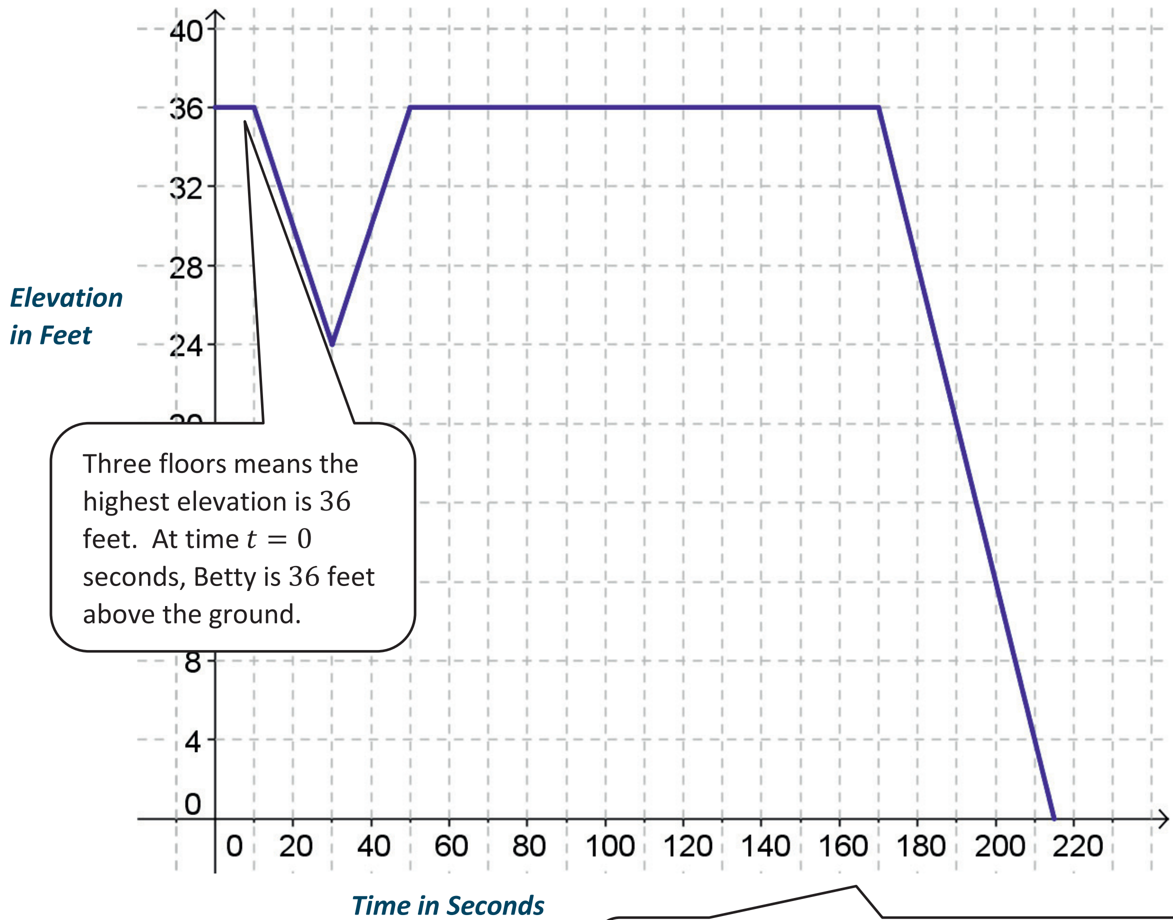
Betty's Elevation-Versus-Time Graph



Since the total time was $10 + 20 + 20 + 120 + 45$ seconds, I need to include up to 215 seconds. I will scale my graph by tens.

2. The graph is a piecewise linear function. Each linear function is defined over an interval of time. List those time intervals.

There are five time intervals measured in seconds: 0 to 10, 10 to 30, 30 to 50, 50 to 170, and 170 to 215.

3. What do the two horizontal line segments on your graph represent?

The horizontal line segments represent the times that

Betty was on the third floor. Her elevation was not changing.

A horizontal line has the same y-coordinates for all points on the graph. This means her elevation stays the same, which happens when she is walking around on the third floor.

I can determine this by finding how much her elevation changed on each time interval and dividing that value by the change in time.

4. What is Betty's average rate of descent from 10 seconds to 30 seconds? From 170 seconds to 215 seconds? How can you use the graph to determine when she was going down the stairs at the fastest average rate?

Average rate of descent from 10 seconds to 30 seconds: $\frac{24-36}{30-10} = -\frac{12}{20} = -\frac{3}{5}$ **ft/sec.** ***This means that, on average, her elevation was decreasing by*** $\frac{3}{5}$ ***of a foot every second.***

Average rate of descent from 170 seconds to 215 seconds: $\frac{0-36}{215-170} = -\frac{36}{45} = -\frac{4}{5}$ **ft/sec.** ***This means that, on average, her elevation was decreasing by*** $\frac{4}{5}$ ***of a foot every second.***

The graph is steeper when she is going down the stairs at a faster average rate.

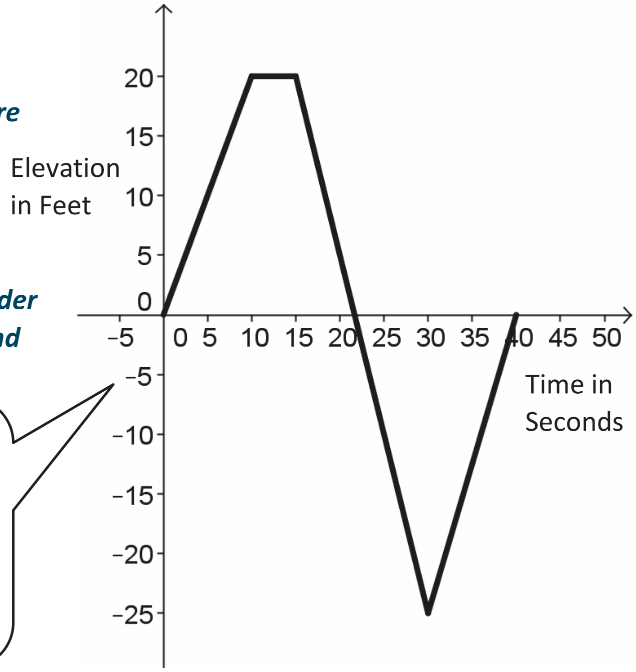
If I measure from the top of her head, all my heights will be 5.5 feet greater than they were originally.

5. If we measured Betty's elevation above the ground from the top of her head (assume she is 5 feet 6 in. tall), how would the graph change?

The whole graph would be translated (shifted) vertically upward 5.5 units.

6. Write a story for the graph of the piecewise linear function shown to the right.

Jens is working on a construction site where they are building a skyscraper. He climbs 20 feet up a ladder in 10 seconds and stays there for 5 seconds. Then he goes down the ladder and keeps going down 25 feet below ground level. At 30 seconds, he immediately climbs back up the ladder at a slightly slower average rate and reaches ground level at 40 seconds.



My story needs to have 4 parts: A part where someone goes above ground level, a part where he stays at the same height between 10 and 15 seconds, a part where he goes below ground level, and a final part where he rises to ground level.

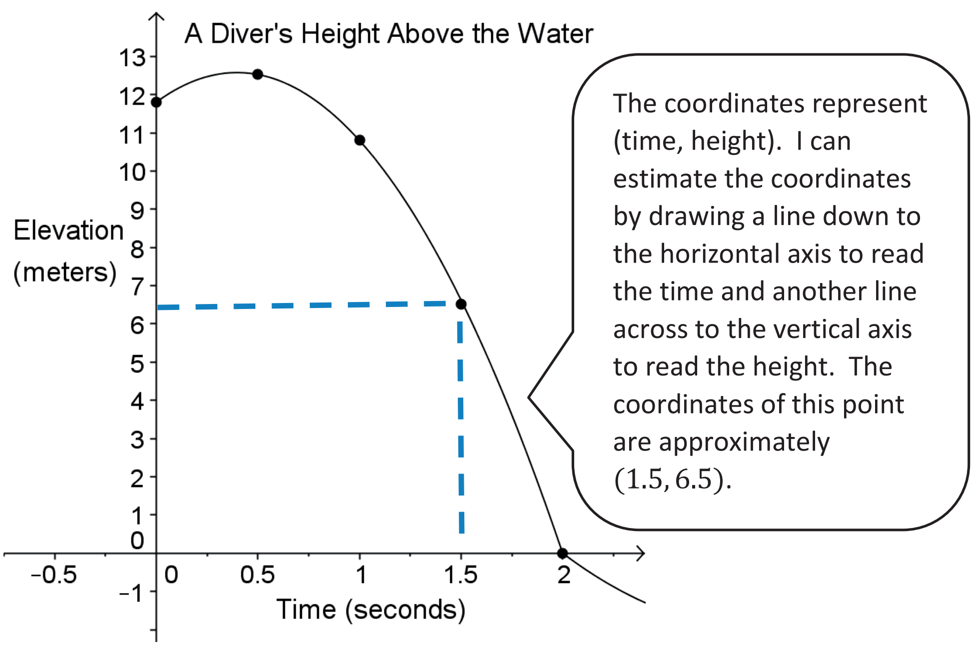
Lesson 2: Graphs of Quadratic Functions

Graphs of Quadratic Functions

Elevation-versus-time graphs that represent relationships, such as a person's elevation as they jump off of a diving board or a ball rolling down a ramp, are graphs of quadratic functions. These types of functions are studied in detail in Module 4.

Analyze the Graph of a Quadratic Function

The elevation-versus-time graph of a diver as she jumps off of a diving board is modeled by the graph shown below. Time is measured in seconds, and the elevation of the top of her head above the water is measured in meters.



Use the information in the graph to answer these questions.

1. What is the height of the diving board? (Assume the diver is 1.5 m tall). Explain how you know.

   When time is 0 seconds, she is on the diving board. The y-coordinate at this point is approximately 11.8 meters. That is the diving board height plus her height. The board is 10.3 meters above the water because $11.8 - 1.5 = 10.3$.

2. When does her head hit the water? Explain how you know.

   The graph represents the elevation of her head above the water. When the y-coordinate is 0, her head will hit the water. The point on the graph is $(2, 0)$. She hits the water after 2 seconds.

3. Estimate her change in elevation in meters from 0 to 0.5 seconds. Also estimate the change in elevation from 1 second to 1.5 seconds.

   From 0 seconds to 0.5 seconds, her elevation changes approximately 0.7 meters because $12.5 - 11.8 = 0.7$. From 1 second to 1.5 seconds, her elevation changes approximately -4.3 meters because $6.5 - 10.8 = -4.3$. The negative sign indicates that she is moving down toward the water on this time interval.

4. Is the diver traveling fastest near the top of her jump or when she hits the water? Use the graph to support your answer.

   The graph appears steeper when she hits the water. The average elevation change between 1.5 seconds and 2 seconds is greater than the elevation change on any other half-second time interval.

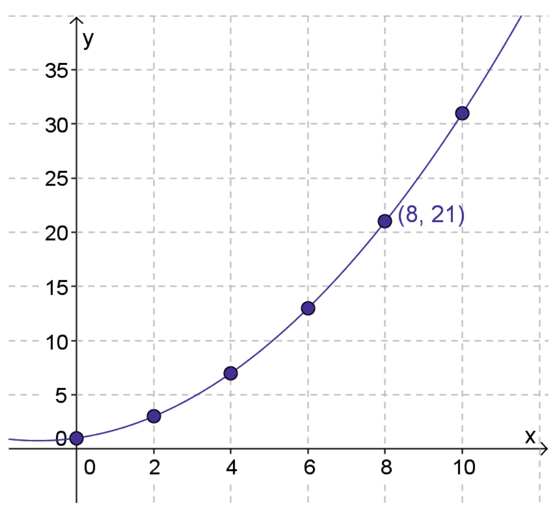
5. Why does the elevation-versus-time graph change its curvature drastically at $t = 2$ seconds?

   When she hits the water, her speed will change because the water is a denser medium than air. This will cause her to slow down instead of speed up.

Examine Consecutive Differences to Find a Pattern and Graph a Quadratic Function

6. Plot the points (x, y) in the table below on a graph (except when x is 8).

x	y
0	1
2	3
4	7
6	13
8	**21**
10	31



When finding the differences in the y-values, I need to subtract the first y-value from the second y-value, the second y-value from the third y-value, and so on.

7. Use the patterns in the differences between consecutive y-values to determine the missing y-value.

   The differences are $3 - 1 = 2$, $7 - 3 = 4$, $13 - 7 = 6$. If the pattern continues, the next differences would be 8 and 10. Adding 8 to 13 gives 21, and adding 10 to that is 31. This missing value is 21.

8. Draw a curve through the points you plotted. Does the curve include the missing y-value?

   Yes, the curve contains the missing y-value.

Lesson 3: Graphs of Exponential Functions

Graphs of Exponential Functions

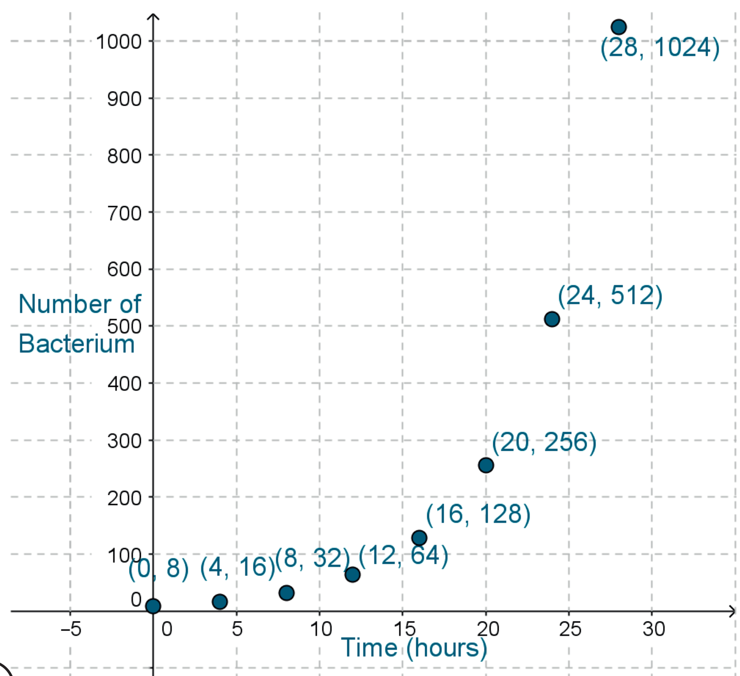
Students construct graphs that represent nonlinear relationships between two quantities, specifically graphs of exponential functions. The growth of bacterium over time can be represented by an exponential function.

Growth of Bacterium

The quantities in this problem are time measured in hours and the number of bacteria present in the sample.

1. A certain type of bacterium doubles every 4 hours. If a samples starts with 8 bacteria, when will there be more than 1000 in the sample? Create a table, and a graph to solve this problem.

Time (hours)	Number of Bacteria
0	**8**
4	**16**
8	**32**
12	**64**
16	**128**
20	**256**
24	**512**
28	**1024**



After 28 hours there will be more than 1000 bacteria in the sample.

I need to extend my table and graph until the number of bacteria is greater than or equal to 1000.

The data are not increasing by equal amounts over equal intervals. From $t = 0$ to $t = 4$, the bacteria increase by 8. From $t = 4$ to $t = 8$, the bacteria increase by 16. The graph of the data will be curved.

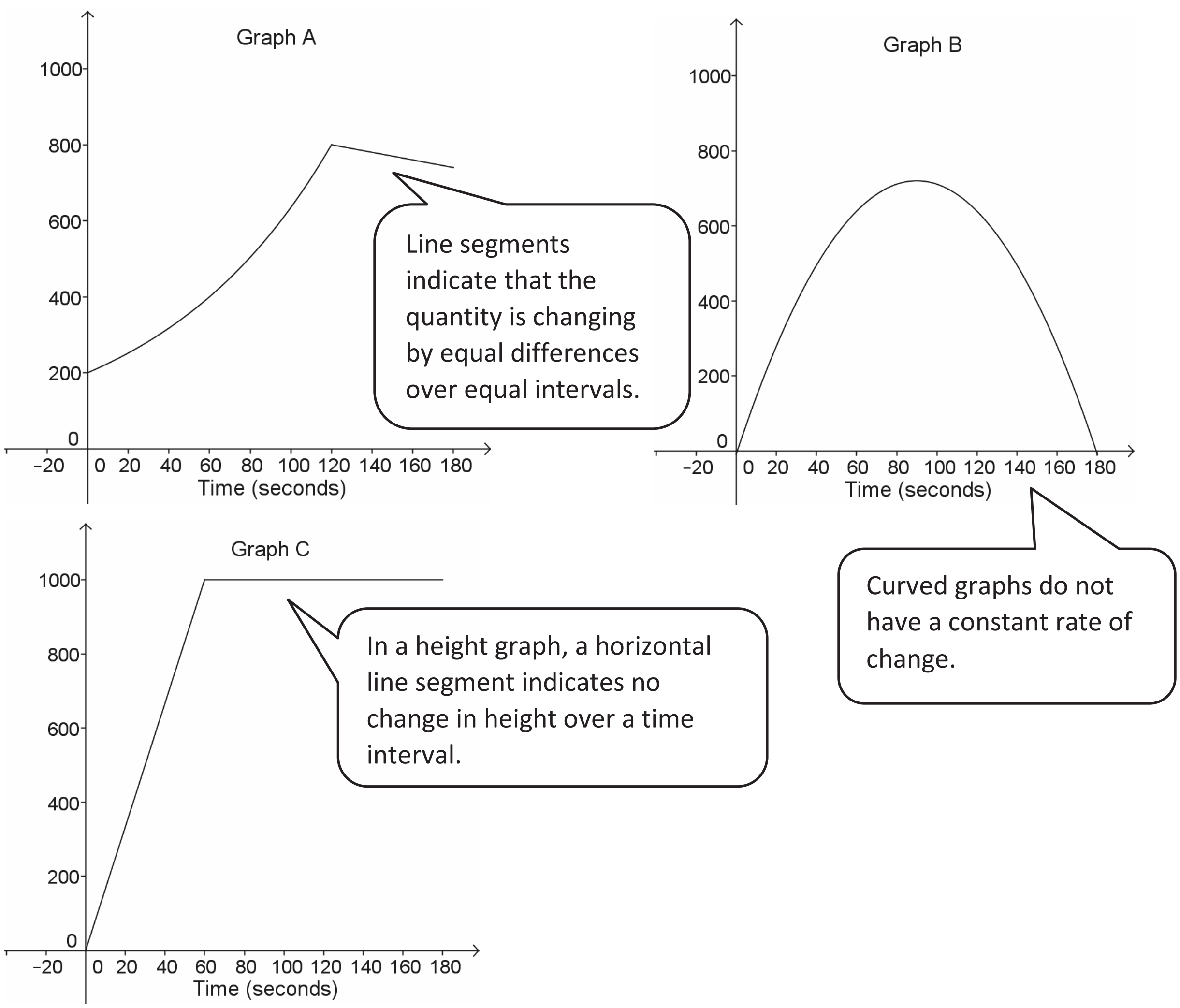
Matching a Story to a Graph

To match a story to a graph, students need to analyze the way that the quantity described in the story is changing as time passes. Be sure to pay attention to any given numerical information to help match the description to a graph.

STORY 1: A plane climbed to an elevation of 1000 feet at a constant speed over a 60-second interval and then maintained that elevation for an additional 2 minutes.

STORY 2: A certain number of bacteria in a petri dish is doubling every minute. After two minutes, a toxin is introduced that stops the growth, and the bacteria start dying. The number of bacteria in the dish decreases by 10 bacteria every 10 seconds.

STORY 3: The physics club launches a rocket into the air. It goes up to a height of over 700 feet and then falls back down to earth in approximately 3 minutes.



2. Match each story to a graph. Explain how you made your choice.

Graph C matches Story 1 because the graph shows a height of 1000 feet after 60 seconds. The height (y-values) stay at 1000 feet for the next 120 seconds. The constant speed means the graph will be composed of line segments, not curves.

Graph A matches Story 2 because the scaling shows a starting amount of 200 bacteria, double that amount after 60 seconds, and then double that amount after 60 more seconds. The line segment shows the fact that the number of bacteria starts decreasing by 10 every 10 seconds.

Graph B matches Story 3 because the rocket is an example of a projectile. The graph is curved because the gravitational constant causes the speed of the rocket to change over time. The function is not a piecewise linear function because the speed is not constant.

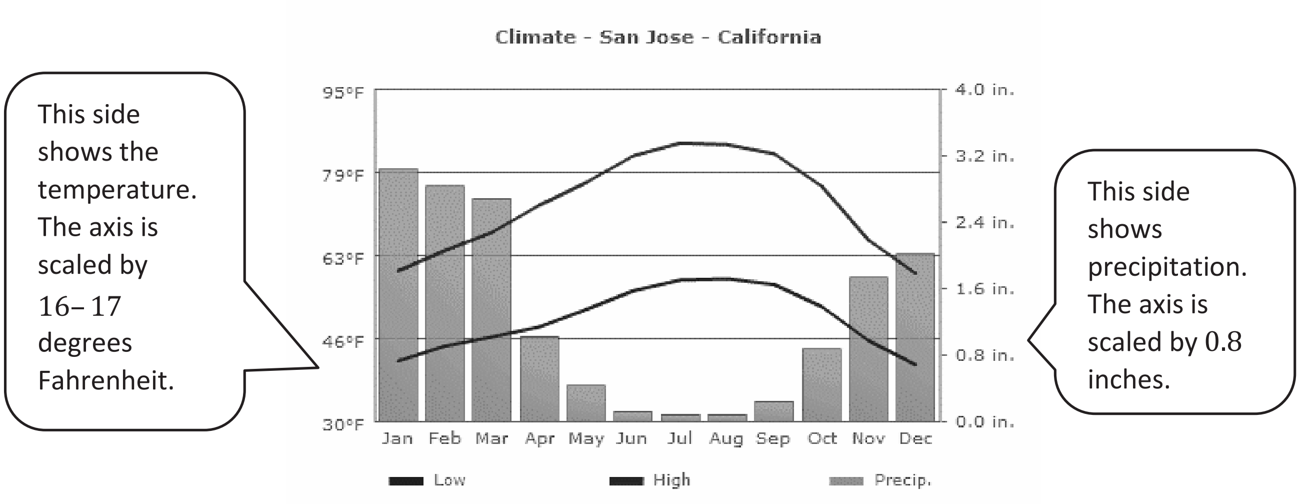
Lesson 4: Analyzing Graphs—Water Usage During a Typical Day at School

Lesson Notes

When analyzing a graph, it is important to identify the quantities represented on the graph and to notice trends in how the quantities change over time.

Analyze a Graph

The following graph shows the monthly average high and low temperature and precipitation in San Jose, California. The temperature scale is on the left side of the graph, and the precipitation scale is on the right side of the graph. Precipitation is shown by the bars on the graph. Temperature is indicated by the lines. Answer the questions following the graph.



1. What is the average low temperature in March? In May? Explain.

   The bottom line crosses the 46°F mark in March, so the average low temperature is 46°F. In May, I need to estimate the height of the bottom line for that month. Drawing a horizontal line to the temperature scale, I can see that the May temperatures are about one-third of the way between 46°F and 63°F. $\frac{1}{3}(63-46)\approx 6$. So the temperature is about $46°\text{F}+6°\text{F}=52°\text{F}$.

2. What is the average precipitation in October? In December? Explain

Looking at the scale on the right, I can see that the bar for October is at 0.8 inches. In December, the bar is about in the middle of 1.6 inches and 2.4 inches, so the average precipitation would be about 2.0 inches.

3. What are the peak months for precipitation? What is the coolest month? What is the warmest month?

It rains the most in January and February. The coolest month is either December or January. Since the average low for December is slightly lower than January, I think December would be the coolest month. The warmest month is July.

4. Do the temperatures vary more in the winter or in the summer months? Explain how you know.

The temperatures vary more in the summer because the distance between the high and low temperatures is greatest in the summer months.

5. How would you describe the weather in the summer months in San Jose? How would you describe the weather in the winter months in San Jose?

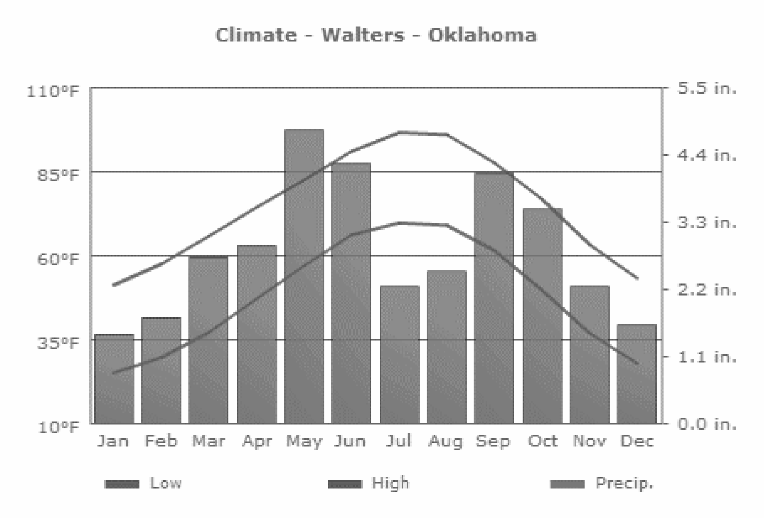
It is warm and dry in the summer. It is cooler in the winter and rains more in the winter months, especially January and February.

6. Do you think it ever snows in San Jose? Explain how you know.

Snow in San Jose would be extremely rare because the average low temperature is above freezing.

7. A climate graph for Walters, Oklahoma, is shown below. How does the weather in this city compare to the weather in San Jose? Identify at least one similarity and two differences.

Both cities are warmest in the summer and coolest in the winter. Walters is warmer in the summer and cooler in the winter than is San Jose. The rainfall peaks in the spring and fall in Walters. In general, there is more rain in Walters than in San Jose. It probably snows in the winter months in Walters because the average low temperature is below $35°\text{F}$***, and the city averages a little over one inch of precipitation per month.***



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Lesson 5: Two Graphing Stories

Lesson Notes

In the problem set, students create graphs and equations of linear functions to represent graphing stories that involve quantities that change at a constant rate. They interpret the meaning of the intersection point of the graphs of two functions.

Create and Interpret Graphs and Equations for Two Stories

Evan and Marla are riding their bicycles south on a road. Marla rides at a constant rate of $\frac{1}{5}$ miles per minute for the first 30 minutes, and then she speeds up to $\frac{1}{4}$ miles per minute. Evan starts 2 miles down the road from Marla and travels at a constant speed of $\frac{1}{4}$ miles per minute. He gets a flat tire after 20 minutes, and it takes him 10 minutes to change his tire. Then he continues riding at a constant speed of $\frac{1}{6}$ miles per minute.

1. Sketch the distance versus time graph for Marla and Evan on a coordinate plane. Start with time 0, and measure the time in minutes.

> Marla's story has two different time intervals. She changes her speed after 30 minutes. I will need two linear equations.

> Evan's story has three different time intervals: when he starts, when he stops to repair his tire, and when he continues riding. I will need three linear equations.

Marla's first change in distance: $\frac{\textbf{1 mi}}{\textbf{5 min}} \times \textbf{30 min} = \textbf{6 mi}$

Marla's second change in distance: $\frac{\textbf{1 mi}}{\textbf{4 min}} \times \textbf{20 min} = \textbf{5 mi}$

Evan's first change in distance: $\frac{\textbf{1 mi}}{\textbf{4 min}} \times \textbf{20 min} = \textbf{5 mi}$

Evan's second change in distance: $\frac{\textbf{0 mi}}{\textbf{1 min}} \times \textbf{10 min} = \textbf{0 mi}$

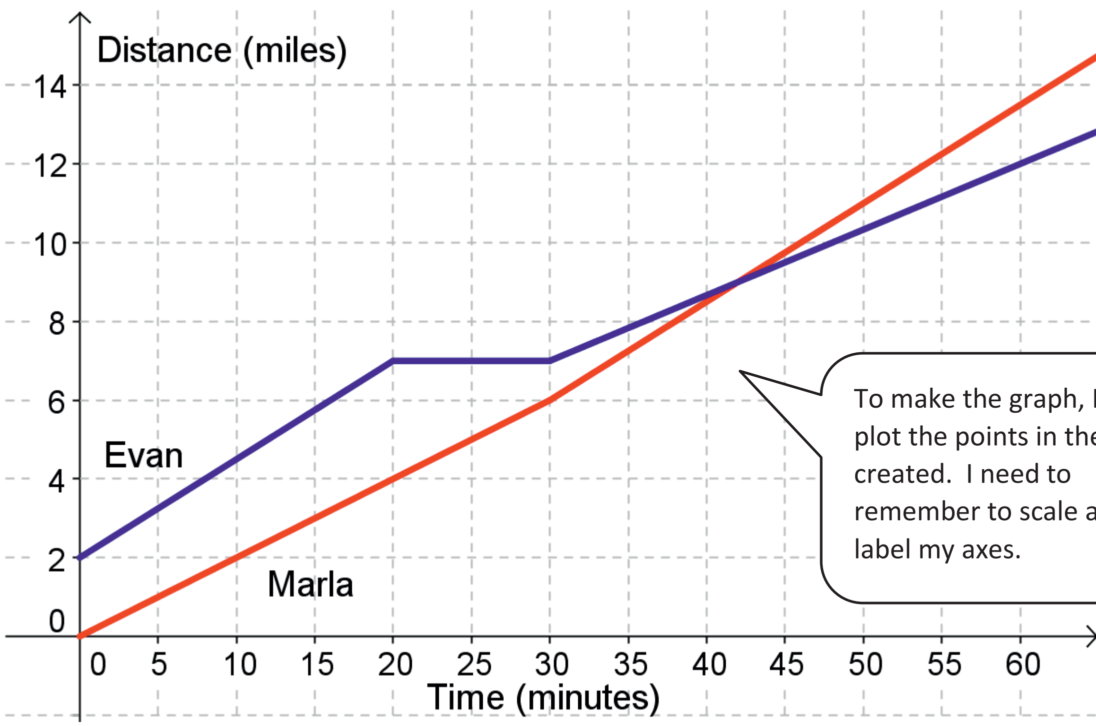
Evan's third change in distance: $\frac{\textbf{1 mi}}{\textbf{6 min}} \times \textbf{30 min} = \textbf{5 mi}$

> Since speed is constant, I can multiply each rider's speed by the time to determine how far they traveled during that interval.

> For the last part of the graph, I can pick a convenient time to get a whole number of miles. Speed is constant, which means the graph is a line segment for each time interval. I just need to find the coordinates of another point to draw the line segment on my graph.

Marla's Coordinates	*Evan's Coordinates*	*Explanation*
$(0, 0)$	$(0, 2)$	*The story is told from Marla's point of view, so she starts at a distance of* 0. *Evan is* 2 *miles down the road.*
$(30, 6)$	$(20, 7)$	*After* 30 *minutes, Marla has traveled* 6 *miles. After* 20 *minutes, Evan has traveled* 5 *miles.* $5 + 2 = 7$
$(50, 11)$	$(30, 7)$	*After* 50 *minutes, Marla has traveled an additional* 5 *miles.* $6 + 5 = 11$ *Evan changes his tire, so his distance stays the same for* 10 *minutes.*
	$(60, 12)$	*After* 60 *minutes, Evan has traveled an additional* 5 *miles.* $7 + 5 = 12$

The coordinates represent (time, distance).



To make the graph, I can plot the points in the table I created. I need to remember to scale and label my axes.

2. Approximately when does Marla pass Evan?

Marla passes Evan at approximately 42 *minutes.*

The intersection point represents the fact that the rider's distances are equal at that time.

3. Create linear equations representing each bicyclist's distance in terms of time (in minutes). You will need three equations for Evan and two equations for Marla.

Let d represent distance (in miles), and let t represent time (in minutes).

I need to state the meaning of my variables.

Evan:

$$d = \frac{1}{4}t + 2, \quad 0 \le t \le 20$$

$$d = 7, \quad 20 < t \le 30$$

$$d = \frac{1}{6}(t - 30) + 7, \quad 30 < t$$

The speed is the slope of the linear equation. The y-intercept is the distance when time $t = 0$.

I need to indicate the time interval for each equation.

Marla:

$$d = \frac{1}{5}t, \quad 0 \le t \le 30$$

$$d = \frac{1}{4}(t - 30) + 6, \quad 30 < t$$

When a line segment with slope m starts at a point (a, b), I can use this form to write the equation: $d = m(t - a) + b$.

4. Use the equations to find the exact coordinates of the point that represents the time and distance when Marla passes Evan on the road.

Use Marla's second equation and Evan's third equation, and set them equal to each other to represent when their distances are the same.

$$\frac{1}{6}(t - 30) + 7 = \frac{1}{4}(t - 30) + 6$$

$$\frac{1}{6}t - 5 + 7 = \frac{1}{4}t - \frac{15}{2} + 6$$ *(Distributive property)*

$$\frac{1}{6}t + 2 = \frac{1}{4}t - \frac{3}{2}$$ *(Combine numbers.)*

$$\frac{1}{6}t + 2 - 2 = \frac{1}{4}t - \frac{3}{2} - 2$$ *(Subtract 2 from both sides.)*

$$\frac{1}{6}t = \frac{1}{4}t - \frac{7}{2}$$

$$\frac{1}{6}t - \frac{1}{4}t = \frac{1}{4}t - \frac{1}{4}t - \frac{7}{2}$$ *(Subtract $\frac{1}{4}t$ from both sides.)*

$$-\frac{1}{12}t = -\frac{7}{2}$$

$$t = \left(-\frac{7}{2}\right)\left(-\frac{12}{1}\right)$$ *(Multiply by the reciprocal of $-\frac{1}{12}$.)*

$$t = 42$$

Using Marla's equation when $t = 42$, calculate the distance at that time. $\frac{1}{4}(42 - 30) + 6 = 9$

The exact coordinates of the intersection point of the two graphs is $(42, 9)$. Marla passes Evan after 42 minutes when she is 9 miles from her starting point.

Lesson 6: Algebraic Expressions—The Distributive Property

Lesson Notes

In this lesson, students begin to explore the structure of expressions first by generating numeric expressions and then by exploring expressions that contain variables.

Using Parentheses to Change the Order of Operations

> I know that any operations inside of parentheses will be evaluated first. I can use parentheses to change the order of the operations.

1. Insert parentheses to make each statement true.

   a. $2 \times 3^2 + 1 + 4 = 23$

   No parentheses are needed here.

   $\mathbf{2 \times 3^2 + 1 + 4 = 2 \times 9 + 1 + 4 = 18 + 1 + 4 = 23}$

   Or the statement could be written as follows:

   $\mathbf{(2 \times 3^2) + 1 + 4 = 23.}$

   > In this case, the parentheses did not change the original order of operations.

   b. $2 \times 3^2 + 1 + 4 = 24$

   $\mathbf{2 \times (3^2 + 1) + 4 = 24}$

   because $\mathbf{2 \times (3^2 + 1) + 4 = 2 \times 10 + 4 = 20 + 4 = 24.}$

   c. $2 \times 3^2 + 1 + 4 = 28$

   $\mathbf{2 \times (3^2 + 1 + 4) = 28}$

   because $\mathbf{2 \times (3^2 + 1 + 4) = 2 \times (9 + 1 + 4) = 2 \times 14 = 28.}$

   d. $2 \times 3^2 + 1 + 4 = 41$

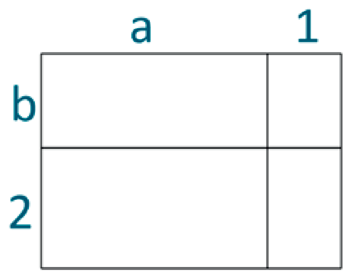
   $\mathbf{(2 \times 3)^2 + 1 + 4 = 41}$

   because $\mathbf{(2 \times 3)^2 + 1 + 4 = 6^2 + 1 + 4 = 36 + 1 + 4 = 41.}$

Drawing a Picture to Represent an Expression

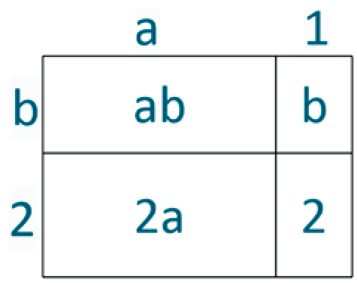
2. Consider the expression $(a+1)(b+2)$.

   a. Draw a picture that represents the expression.



I can think of the factors in the product as being the sides of a rectangle.

   b. Write an equivalent expression by applying the distributive property.

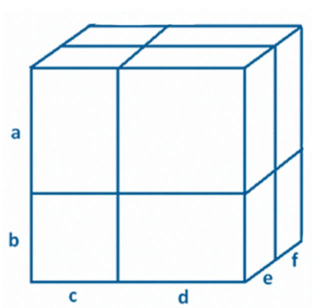


$$(\boldsymbol{a}+\mathbf{1})(\boldsymbol{b}+\mathbf{2}) = \boldsymbol{ab}+\mathbf{2}\boldsymbol{a}+\boldsymbol{b}+\mathbf{2}$$

I can find an equivalent expression by adding the areas of the 4 rectangles.

3. Consider the expression $(a+b)(c+d)(e+f)$.

   a. Draw a picture that represents the expression.



   b. Write an equivalent expression by applying the distributive property.

I can first use the distributive property to rewrite the expression $(c+d)(e+f)$.

$$\begin{aligned}(\boldsymbol{a}+\boldsymbol{b})(\boldsymbol{c}+\boldsymbol{d})(\boldsymbol{e}+\boldsymbol{f}) &= (\boldsymbol{a}+\boldsymbol{b})[(\boldsymbol{c}+\boldsymbol{d})(\boldsymbol{e})+(\boldsymbol{c}+\boldsymbol{d})(\boldsymbol{f})]\\ &= (\boldsymbol{a}+\boldsymbol{b})(\boldsymbol{ce}+\boldsymbol{de}+\boldsymbol{cf}+\boldsymbol{df})\\ &= (\boldsymbol{a}+\boldsymbol{b})\boldsymbol{ce}+(\boldsymbol{a}+\boldsymbol{b})\boldsymbol{de}+(\boldsymbol{a}+\boldsymbol{b})\boldsymbol{cf}+(\boldsymbol{a}+\boldsymbol{b})\boldsymbol{df}\\ &= \boldsymbol{ace}+\boldsymbol{bce}+\boldsymbol{ade}+\boldsymbol{bde}+\boldsymbol{acf}+\boldsymbol{bcf}+\boldsymbol{adf}+\boldsymbol{bdf}\end{aligned}$$

I can continue using the distributive property until there are no longer any parentheses in the expression.

Lesson 7: Algebraic Expressions—The Commutative and Associative Properties

Lesson Notes

In this lesson, students continue to explore the structure of expressions by applying the commutative and associative properties to algebraic expressions along with the distributive property, which was discussed in Lesson 6. Geometric models and flow diagrams are used to aid students in writing equivalent expressions.

Constructing a Mathematical Proof to Verify that Two Expressions are Equivalent

1. Write a mathematical proof to show that $(x+2)^2$ is equivalent to x^2+4x+4.

I could also use the commutative property to rewrite this expression as $x(x+2)+2(x+2)$.

$(x+2)^2 = (x+2)(x+2)$

$= (x+2)x + (x+2)2$ ***distributive property***

$= x^2 + 2x + 2x + 4$ ***distributive property***

$= x^2 + (2x+2x) + 4$ ***associative property***

$= x^2 + 4x + 4$

I first need to write this as a product. Note that $(a+b)^2 \neq a^2+b^2$. This is a common mistake that I disproved in Lesson 6.

Using Arithmetic Properties to Evaluate Expressions

Use the commutative and associative properties to rewrite each expression so that it can be easily evaluated using mental math. Then, find the value of the expression.

2. $81+125+9+75$

$$81+9+125+75 = (81+9)+(125+75) = 90+200 = 290$$

This is the commutative property of addition.

This is the associative property of addition.

I should apply the commutative and associative properties of multiplication.

3. $81 \times 125 \times \frac{1}{9} \times \frac{1}{25}$

$$\mathbf{81 \times \frac{1}{9} \times 125 \times \frac{1}{25} = \left(81 \times \frac{1}{9}\right) \times \left(125 \times \frac{1}{25}\right) = 9 \times 5 = 45}$$

Properties of Exponents

4. Replace the expression $\left((4a^{12})(2a^{-3})^3\right) \div (16a^4)$ with an equivalent expression in which the variable of the expression appears only once with a positive number for its exponent. (For example, $\frac{7}{b^2} \cdot b^{-4}$ is equivalent to $\frac{7}{b^6}$.)

$$\left((4a^{12})(2a^{-3})^3\right) \div (16a^4) = \left((4a^{12})(2^3a^{-9})\right) \div (16a^4)$$
$$= (4 \cdot 8 \cdot a^{12} \cdot a^{-9}) \div (16a^4)$$
$$= (32a^3) \div (16a^4)$$
$$= \frac{32a^3}{16a^4}$$
$$= \frac{2}{a}$$

I recall that b^{-4} is equivalent to $\frac{1}{b^4}$, so $\frac{7}{b^2} \cdot b^{-4} = \frac{7}{b^2} \cdot \frac{1}{b^4}$ 7

Lesson 8: Adding and Subtracting Polynomials

Vocabulary Associated with Polynomial Expressions

1. Give an example of a monomial that has a degree of 5.

   I can only use the operation of multiplication to create a monomial. The exponents on the variable symbols must add to 5.

   Sample answers:

   $\boldsymbol{4x^5}$

   $\boldsymbol{a^2b^3}$

   $\boldsymbol{3mn^4}$

2. Give an example of a binomial that has a degree of 5.

   I need to add two monomials and make sure that one of them has a degree of 5 and the other one has a lower degree. I can use subtraction if I select a term with a negative coefficient.

   Sample answers:

   $\boldsymbol{4x^5 + 2}$

   $\boldsymbol{a^2b^3 - 5ab}$

   $\boldsymbol{3mn^4 + 3m}$

3. Betty says that $a^2b^3 + 7a^2b^3$ is a monomial in disguise. Is she correct in saying this?

   Yes, she is correct. Since a^2b^3 and $7a^2b^3$ are like terms, they can be combined into a single term.

   Therefore, $a^2b^3 + 7a^2b^3$ is equivalent to the monomial $8a^2b^3$.

4. Bobbie says that $a^2b^3 + 7a^3b^2$ is a monomial in disguise. Is she correct in saying this?

   No, she is incorrect. a^2b^3 and $7a^3b^2$ are not like terms and cannot be combined into a single term. This expression is a binomial and cannot be expressed as a monomial.

Adding and Subtracting Polynomials

Find each sum or difference by combining the parts that are alike.

5. $(4a^2 + 6a + 7) + (2a^2 + 8a + 10)$

$$\begin{aligned}(4a^2 + 6a + 7) + (2a^2 + 8a + 10) &= (4a^2 + 2a^2) + (6a + 8a) + (7 + 10) \\ &= 6a^2 + 14a + 17\end{aligned}$$

I want to write my final answer in standard form, which means I start with the highest degreed monomial and then write the remaining terms in descending order.

6. $(10k - 2k^5 - 14k^4) - (12k^5 - 6k - 10k^3)$

$$\begin{aligned}(10k - 2k^5 - 14k^4) - (12k^5 - 6k - 10k^3) &= 10k - 2k^5 - 14k^4 - 12k^5 + 6k + 10k^3 \\ &= (-2k^5 - 12k^5) - 14k^4 + 10k^3 + (10k + 6k) \\ &= -14k^5 - 14k^4 + 10k^3 + 16k\end{aligned}$$

I am using the distributive, associative, and commutative properties from Lessons 6 and 7 to rewrite this expression.

7. $(10x^2 - 8) - 2(4x^2 - 14) + 6(x^2 + 3)$

$$\begin{aligned}(10x^2 - 8) - 2(4x^2 - 14) + 6(x^2 + 3) &= 10x^2 - 8 - 8x^2 + 28 + 6x^2 + 18 \\ &= (10x^2 - 8x^2 + 6x^2) + (-8 + 28 + 18) \\ &= 8x^2 + 38\end{aligned}$$

Lesson 9: Multiplying Polynomials

In Lesson 8, I learned that standard form means to start with the highest degreed monomial and then write the remaining terms in descending order.

Multiplying Polynomials

Use the distributive property to write each of the following expressions as a polynomial in standard form.

1. $(3x-1)(2x-5)$

$$\begin{aligned}(3x-1)(2x-5) &= 3x(2x-5)-1(2x-5)\\ &= 6x^2-15x-2x+5\\ &= 6x^2-17x+5\end{aligned}$$

I start by distributing the second binomial to each term in the first binomial.

I can use my answer from Problem 1.

2. $x(3x-1)(2x-5)$

$$\begin{aligned}x(3x-1)(2x-5) &= x(6x^2-17x+5)\\ &= 6x^3-17x^2+5x\end{aligned}$$

I can use the commutative property to rearrange the factors in this expression and then use my answer from Problem 1 again.

3. $(3x-1)(x+2)(2x-5)$

$$\begin{aligned}(3x-1)(x+2)(2x-5) &= (x+2)(3x-1)(2x-5)\\ &= (x+2)(6x^2-17x+5)\\ &= x(6x^2-17x+5)+2(6x^2-17x+5)\\ &= 6x^3-17x^2+5x+12x^2-34x+10\\ &= 6x^3-5x^2-29x+10\end{aligned}$$

4. $(4a^2 + 6a + 7)(2a^2 + 8a + 10)$

$$\begin{aligned}(4a^2 + 6a + 7)(2a^2 + 8a + 10) &= 4a^2(2a^2 + 8a + 10) + 6a(2a^2 + 8a + 10) + 7(2a^2 + 8a + 10)\\ &= 8a^4 + 32a^3 + 40a^2 + 12a^3 + 48a^2 + 60a + 14a^2 + 56a + 70\\ &= 8a^4 + 44a^3 + 102a^2 + 116a + 70\end{aligned}$$

First, I need to write this expression as a product.

5. $(k + 1)^3$

$$\begin{aligned}(k+1)^3 &= (k+1)(k+1)(k+1)\\ &= (k+1)(k(k+1) + 1(k+1))\\ &= (k+1)(k^2 + k + k + 1)\\ &= (k+1)(k^2 + 2k + 1)\\ &= k(k^2 + 2k + 1) + 1(k^2 + 2k + 1)\\ &= k^3 + 2k^2 + k + k^2 + 2k + 1\\ &= k^3 + 3k^2 + 3k + 1\end{aligned}$$

Lesson 10: True and False Equations

Lesson Notes

This lesson focuses on understanding that an equation is a statement of equality between two expressions. An equation is not true or false until we assign a value to the variable. An equation is true when a value assigned to the variable makes both expressions evaluate to the same number; otherwise it is false.

True or False?

Determine whether the following number sentences are true or false.

1. $3 + \frac{5}{2} = 2\left(\frac{11}{4}\right)$

   TRUE. Both expressions evaluate to $\mathbf{5.5}$.

> Do both sides evaluate to the same number? If yes, then the equation is true. Otherwise, it is false. I should be able to do the arithmetic. I can also check my work with a calculator.

2. $1 - 1 + 1 - 1 + 1 - 1 + 1 = (-1)^7$

   FALSE. The left expression is equal to $\mathbf{1}$, and the right expression is equal to $\mathbf{-1}$.

In the following equations, let $x = -2$ and let $y = 3.5$. Determine whether the following equations are true, false, or neither.

> I need to substitute the given values for the variables and see if both sides evaluate to the same number.

3. $xy + 1 = 6$

   Substituting $\mathbf{-2}$ for x and $\mathbf{3.5}$ for y gives the expression $\mathbf{(-2)(3.5) + 1}$, which evaluates to $\mathbf{-7 + 1 = -6}$.

   This number is NOT the same as the number on the right side of the equation.

   This equation is FALSE for these values of x and y.

4. $x + 2y = 5$

   Substituting $\mathbf{-2}$ for x and $\mathbf{3.5}$ for y gives the expression $\mathbf{-2 + 2(3.5)}$, which evaluates to $\mathbf{2 + 7 = 5}$.

   This number is the same as the number on the right side. This equation is TRUE for these values of x and y.

5. $z + 3 = x$

Substituting -2 for x gives the equation $z + 3 = -2$.

This equation is neither true nor false when x is -2 because we have not assigned a value to z.

I might be able to use equation solving skills I learned in previous grades, OR I can guess and check to see which values of the variable make both sides equal the same number.

Make an Equation a True Statement

For each equation, assign a value to the variable to make the equation a true statement.

6. $7x - 10 = 60$

$x = 10$ because $7(10) - 10$ evaluates to 60.

7. $(x^2 + 1)(x^2 - 4) = 0$

$x = 2$ or $x = -2$

Both values work because $\left((2)^2 + 1\right)\left((2)^2 - 4\right) = 0$

and $\left((-2)^2 + 1\right)\left((-2)^2 - 4\right) = 0$.

Since we are multiplying two expressions, the left side of the equation will equal 0 if either expression is 0 because $0 \cdot$ any number $= 0$. I can see that 2 works because $2^2 = 4$. So does -2 because $(-2)^2 = 4$.

8. $x^2 = -36$ for ________.

There is no value of x that makes the equation true because a number multiplied by itself is always positive.

I need to think of a number that when squared is 9. That would be 3 or -3. From there I can figure out a value for y.

9. $(y + 5)^2 = 9$ for ___________.

$y = -8$ or $y = -2$

Substituting -8 for y into the left side gives $(-8 + 5)^2$. This expression is equal to 9.

Substituting -2 for y into the left side gives $(-2 + 5)^2$, which also evaluates to 9.

10. $(x + 4)^2 = x^2 + 4^2$

$x = 0$

I think 0 is the only number that works. When you add two numbers and then square the result, you get a different value than you do when you square the numbers first and then add them.

11. $\frac{1+x}{2+x} = \frac{5}{6}$ if __________.

$x = 4$ because

$\frac{1+4}{2+4} = \frac{5}{6}$.

12. The area of a circle of radius r is 9π square inches when ___________.

$r = 3$ inches because

$\pi(3)^2 = 9\pi$ using the area formula for a circle.

I know the area formula for a circle is $A = \pi r^2$ where A is the area and r is the radius.

Lesson 11: Solution Sets for Equations and Inequalities

Lesson Notes

The solution set of an equation can be described in words, using set notation, or graphically on a number line. When solving an inequality, we also find the solution set.

Examples of Set Notation

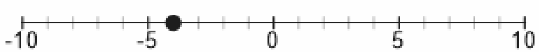
Words	**Set Notation**
The set of real numbers equal to 1, 2, or 3	$\{1, 2, 3\}$
The set of all real numbers greater than 2	$\{x \text{ real} \mid x > 2\}$
The null set	$\{\ \ \}$

The null set contains no numbers. An equation with no solutions has a solution set that is called the null set.

Describe the Solution Set Given a Graph

For each solution set graphed below, (a) describe the solution set in words, (b) describe the solution set in set notation, and (c) write an equation or an inequality that has the given solution set.

1



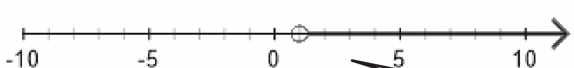
(a) The set of real numbers equal to -4.

(b) (b) $\{-4\}$ (c) $x = -4$

The easiest equation is just $x =$ a number. I could make a more complicated equation by applying a property and adding 3 to both sides.

$x + 3 = -4 + 3$

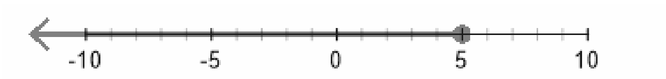
2.



(a) The set of real numbers greater than 1.

(b) $\{x \text{ real} \mid x > 1\}$ (c) $x > 1$

When the endpoint is an open circle, I use a strict inequality ($<$ or $>$). A filled in circle is used for $\geq$ or $\leq$.

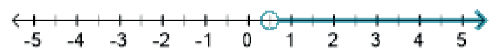
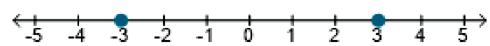
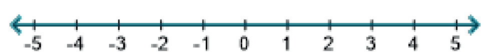
3. 

(a) The set of real numbers less than or equal to 5.

(b) $\{x \text{ real} \mid x \leq 5\}$ ***(c)*** $x \leq 5$

Describe the Solution Set Given an Equation or Inequality

For each equation, describe the solution set in words, in set notation, and using a graph.

	Solution Set in Words	Solution Set in Set Notation	Graph
4. $x > \frac{1}{2}$	***The set of real numbers greater than*** $\frac{1}{2}$	$\left\{x \text{ real} \mid x > \frac{1}{2}\right\}$	
5. $x^2 = 9$	***The set of real numbers equal to*** 3 ***or*** -3	$\{-3, 3\}$	
6. $3x + 3 = 3(x + 1)$	***The set of all real numbers***	$\mathbb{R}$	

Decide If Two Expressions Are Equivalent

Two expressions are equivalent if they create an equation whose solution set is all real numbers for which each expression is defined.

7. Are the expressions algebraically equivalent? If yes, state the property. If not, state why, and change the equation to create an equation whose solution set is all real numbers.

   a. $2(x - 3) = 2x - 6$

   Yes. By the distributive property, $2(x - 3) = 2x - 6$.

   b. $x + 3x + 4x = 8x^3$

   No. The left side is equal to $8x$ ***not*** $8x^3$. ***An equation whose solution set is all real numbers would be*** $x + 3x + 4x = 8x$.

c. $x - 3 = 3 - x$

No. Subtraction is not commutative. An equation whose solution set is all real numbers would be $\boldsymbol{x - 3 = -3 + x}$***.***

Lesson 12: Solving Equations

Equations with the Same Solution Set

1. Which of the following equations have the same solution set? Give a reason for your answer that does not depend on solving the equation.

   a. $2x - 1 = 4x + 6$

   b. $1 - 2x = x - 3$

   c. $4x - 2 = 8x + 12$

   d. $-\frac{1}{2} + x = 2x + 3$

   e. $1 - 3x = -3$

I should be trying to identify properties that were used to transform an equation into a new one without changing the solution set.

Equations (a), (c), and (d) are the same. Equation (a) is the same as Equation (c) after multiplying both sides of the equation by 2. Equation (d) is the same as Equation (a) after dividing both sides of the equation by 2 and rewriting the left side using the commutative property of addition.

$$\mathbf{2x - 1 = 4x + 6}$$
$$\mathbf{\frac{2x}{2} - \frac{1}{2} = \frac{4x}{2} + \frac{6}{2}}$$
$$\mathbf{x - \frac{1}{2} = 2x + 3}$$
$$\mathbf{-\frac{1}{2} + x = 2x + 3}$$

I can check to see if Equations (a) and (c) are the same by applying properties. Divide by 2, and rewrite each side. Rearrange the left side using the commutative property.

Equations (b) and (e) are the same. Equation (e) is the same as Equation (b) after subtracting x from both sides.

Solve Equations

Solve the equation, check your solution, and then graph the solution set.

2. $3.5(x - 2) = x - 9$

$\mathbf{3.5x - 7 = x - 9}$	***Rewrite using distributive property.***
$\mathbf{3.5x - 7 - x = x - 9 - x}$	***Subtract x from both sides of the equation.***
$\mathbf{2.5x - 7 = -9}$	***Rewrite by combining like terms.***
$\mathbf{2.5x - 7 + 7 = -9 + 7}$	***Add 7 to both sides of the equation.***
$\mathbf{2.5x = -2}$	***Rewrite by combining like terms.***
$\mathbf{\frac{2.5x}{2.5} = \frac{-2}{2.5}}$	***Divide both sides of the equation by 2.5.***
$\mathbf{x = -0.8}$	

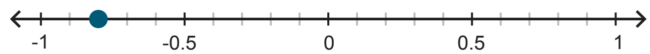
The solution set is $\{-0.8\}$.

Check the solution.

Is this a true number sentence?

$$\mathbf{3.5(-0.8-2)=-0.8-9}$$
$$\mathbf{3.5(-2.8)=-9.8}$$
$$\mathbf{-9.8=-9.8}$$

Graph of the solution set:



The distributive property can be applied to the left side because $x+3$ represents a real number. I could also make a diagram like we did in Lesson 6 to rewrite the left side.

3. $(x+3)(x-2)=x^2+x-9$

$\mathbf{(x+3)x+(x+3)(-2)=x^2+x-9}$	***Distributive property***
$\mathbf{x^2+3x-2x-6=x^2+x-9}$	***Distributive property and combine like terms***
$\mathbf{x^2+x-6=x^2+x-9}$	
$\mathbf{x^2+x-6-x^2-x=x^2+x-9-x^2-x}$	***Subtract x^2 and x from both sides of the equation.***
$\mathbf{-6=-9}$	

This equation is never true.

The solution set is $\{\ \ \}$.

No graph is needed since there are no solutions.

When solving an equation results in a false number sentence, the original equation has no solution.

I learned about the null set in Lesson 11.

Lesson 13: Some Potential Dangers When Solving Equations

If an equation has a solution, as long as I apply the properties without making any errors, each new equation I write will have the same solution as the previous ones.

Solving Equations and Explaining Your Work

Solve the equation for x. For each step, describe the operation used to convert the equation. How do you know that the initial equation and the final equation have the same solution set?

1. $2x(x-3)-(x^2-9)=\frac{1}{8}(8x^2-16x+32)$

$\mathbf{2x^2-6x-x^2+9=x^2-2x+4}$	***Distributive property***
$\mathbf{x^2-6x+9=x^2-2x+4}$	***Commutative property/combine like terms***
$\mathbf{-6x+9=-2x+4}$	***Subtract x^2 from both sides.***
$\mathbf{-4x+9=4}$	***Add $2x$ to both sides.***
$\mathbf{-4x=-5}$	***Subtract 9 from both sides.***
$\mathbf{x=\frac{5}{4}}$	***Divide both sides by -4.***

The solution set is $\left\{\frac{5}{4}\right\}$. The final equation $x=\frac{5}{4}$ has the same solution set as the initial equation because both equations are true number sentences when I substitute $\frac{5}{4}$ for x.

I can check my solution by substituting it into the original equation to see if it makes a true number sentence.

Some Operations Create Extra Solutions

2. Consider the equation $2x-1=5$.

   a. What is the solution set?

   $$\mathbf{2x-1=5}$$
   $$\mathbf{2x=6}$$
   $$\mathbf{x=3}$$

   The solution set is $\{3\}$.

   b. Multiply both sides by x. What is the new solution set?

   $$\mathbf{x(2x-1)=5x}$$

   The solution set is $\{0,3\}$.

3 is still a solution. When x is assigned the value 3, I am just multiplying both sides by 3. When x is assigned the value 0, I also get a true number sentence.

$$0(2\cdot 0-1)=5\cdot 0$$

c. Multiply both sides by $(x-1)$. What is the new solution set?

$\mathbf{(x-1)(2x-1)=(x-1)(5)}$

The solution set is $\mathbf{\{1,3\}}$.

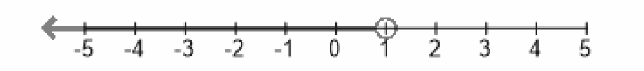
1 is a solution because substituting 1 for x makes both sides equal 0.

Lesson 14: Solving Inequalities

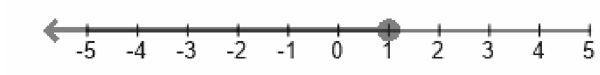
Lesson Notes

The properties of inequality are if-then moves we can apply to solve an inequality. We graph the solution set on a number line.

- When graphing a strict inequality like $x < 1$, use an open circle at the endpoint.



- When graphing an inequality like $x \leq 1$, use a filled-in circle at the endpoint.



Find and Graph the Solution Set of an Inequality

I learned how to write set notation in Lesson 11. I graphed solution sets in this lesson as well.

Find the solution set. Express the solution in set notation and graphically on the number line.

1. $-2(x-8) \leq 6$

$\mathbf{-2x + 16 \leq 6}$

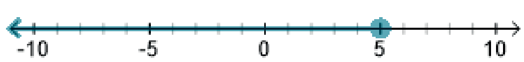
$\mathbf{16 \leq 6 + 2x}$

$\mathbf{10 \leq 2x}$

$\mathbf{5 \leq x}$

I use the distributive property on the left side and then the addition property of inequality to add $2x$ to both sides. Then, I subtract 6 from both sides and divide by 2. The inequality $5 \leq x$ is the same as the inequality $x \geq 5$.

The solution set is $\{x \text{ real} \mid x \leq 5\}$.



I need to fill in the point at 5 and draw a ray to the left of 5 on the number line.

2. $2x + \frac{7}{2} \neq 3$

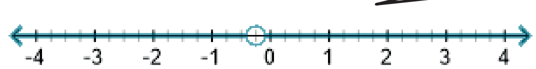
$\mathbf{2x \neq -\frac{1}{2}}$

$\mathbf{x \neq -\frac{1}{4}}$

My solution is all real numbers except for $-\frac{1}{4}$.

The solution set is $\left\{x \text{ real} \mid x \neq -\frac{1}{4}\right\}$.

I need an open circle at $-\frac{1}{4}$, and I need to shade the rest of the number line.



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Lesson 15: Solution Sets of Two or More Equations (or Inequalities) Joined by "And" or "Or"

Representing Inequalities Joined by "And" or "Or"

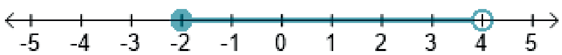
Consider the inequality $-2 \leq x < 4$.

When written in this fashion, the inequalities are joined by an "and."

1. Write the inequality as a compound sentence.

   $x \geq -2$ and $x < 4$

2. Graph the inequality on a number line.



The solution to inequalities joined by an "and" show the portion of the number line where *both* inequalities are true.

3. How many solutions are there to the inequality? Explain.

   There are an infinite number of solutions. x can be any value between -2 and 4 including -2. This includes integer and non-integer values like $\frac{1}{2}$ or $-\frac{1}{10}$ or $\sqrt{2}$. This set of numbers is infinite.

4. What are the largest and smallest possible values for x? Explain.

   The smallest value is -2. There is no largest value because x can get infinitely close to 4 but cannot equal 4.

Writing Inequalities Given a Sentence

Write a single or compound inequality for each scenario.

I know that "at least" means to include the 500 in my inequality.

5. There is at least $\$500$ in my savings account.

   Let x represent the number of dollars in the savings account. $x \geq 500$.

I need to define the meaning of my variable and then write the inequality or inequalities.

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6. A cell phone plan includes between 0 and 800 minutes for free.

Let x represent the number of free minutes. $0 < x < 800$.

I also could have used two inequalities joined by "and": $x > 0$ and $x < 800$.

Solve and Graph the Solution(s)

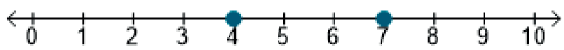
7. $x - 9 = -5$ or $2x - 5 = 9$

$$\mathbf{x - 9 = -5}$$
$$\mathbf{x = 4}$$

or

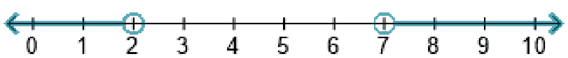
$$\mathbf{2x - 5 = 9}$$
$$\mathbf{2x = 14}$$
$$\mathbf{x = 7}$$

I need to make a number line and plot a point for each solution since these equations are joined by an "or."



8. $x > 7$ or $x < 2$

The endpoints are not included, so I do not fill in the points.

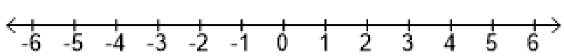


9. $x > 3$ and $2x + 11 = 5$

I can have an inequality and an equation joined by an "and" or an "or."

$$\mathbf{2x + 11 = 5}$$
$$\mathbf{2x = -6}$$
$$\mathbf{x = -3}$$

The solution is the null set because there are no numbers greater than 3 and at the same time equal to -3.



Lesson 16: Solving and Graphing Inequalities Joined by "And" or "Or"

Solve each inequality for x, and graph the solution on a number line.

1. $8(x-3) > 16$ or $8x + 5 < 3x$

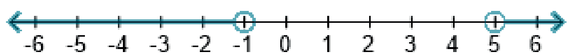
> I need to solve each inequality in the compound sentence and then graph the solution set.

$$\mathbf{8(x-3) > 16}$$
$$\mathbf{8x - 24 > 16}$$
$$\mathbf{8x > 40}$$
$$\mathbf{x > 5}$$

or

$$\mathbf{8x + 5 < 3x}$$
$$\mathbf{8x < 3x - 5}$$
$$\mathbf{5x < -5}$$
$$\mathbf{x < -1}$$

Solution: $\{x \text{ real} \mid x < -1 \textit{ or } x > 5\}$



> When a number is a solution to inequalities joined by "or," the solution can be part of either inequality.

> When I draw the number line, I need to make sure it includes enough room to show both inequalities.

2. $-8 \le \frac{2}{3}x - 4 \le 2$

> In Lesson 15, I learned that this inequality is a compound sentence joined by an "and."

$\mathbf{-8 \le \frac{2}{3}x - 4}$ ***and*** $\mathbf{\frac{2}{3}x - 4 \le 2}$

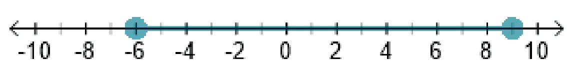
$$\mathbf{-8 \le \frac{2}{3}x - 4}$$
$$\mathbf{-4 \le \frac{2}{3}x}$$
$$\mathbf{\frac{3}{2}(-4) \le x}$$
$$\mathbf{-6 \le x}$$

> After adding 4 to both sides, I can multiply both sides by the reciprocal of $\frac{2}{3}$.

and

$$\mathbf{\frac{2}{3}x - 4 \le 2}$$
$$\mathbf{\frac{2}{3}x \le 6}$$
$$\mathbf{x \le (6)\frac{3}{2}}$$
$$\mathbf{x \le 9}$$

> When inequalities are joined by an "and," the graph is only the set of numbers that satisfy both inequalities at the same time.



Solution: $\{x \text{ real} \mid x \ge -6 \text{ and } x \le 9\}$

Lesson 17: Equations Involving Factored Expressions

Solving Equations Using the Zero-Product Property

Find the solution set of each equation.

1. $(x-3)(2x+1)(x+100)=0$

   $\boldsymbol{x-3=0}$ ***or*** $\boldsymbol{2x+1=0}$ ***or*** $\boldsymbol{x+100=0}$

   > By the zero-product property, a product will equal 0 only when one of its factors is equal to 0. I can rewrite the original equation as a compound sentence joined by an "or."

   The solutions to these three equations are $\boldsymbol{3, -\frac{1}{2}}$***, and*** $\boldsymbol{-100}$***, respectively.***

   Solution set: $\boldsymbol{\left\{-100, -\frac{1}{2}, 3\right\}}$

   > The proper way to write a solution to an equation is using set notation.

2. $x(x-3)+7(x-3)=0$

   > I need to rewrite this equation using properties of algebra so one side is a product, and the other side is 0.

   $\boldsymbol{(x-3)(x+7)=0}$

   $\boldsymbol{x-3=0}$ **or** $\boldsymbol{x+7=0}$

   The solution to the first equation is $\boldsymbol{3}$***.***

   The solution to the second equation is $\boldsymbol{-7}$***.***

   Solution set: $\boldsymbol{\{-7, 3\}}$

3. $2x^2+4x=0$

   $\boldsymbol{2x\cdot x+2x\cdot 2=0}$
   $\boldsymbol{2x(x+2)=0}$

   > I can rewrite expressions using the distributive property: $a(b+c)=ab+ac$.

   $\boldsymbol{2x=0}$ ***or*** $\boldsymbol{x+2=0}$

   Solution set: $\boldsymbol{\{0, -2\}}$

4. $x(x-3)=5x$

   > I can only divide both sides by x as long as I assume x does not equal 0. However, the number 0 is a solution. I can see that by substituting into the original equation.

   $\boldsymbol{x(x-3)-5x=0}$
   $\boldsymbol{x(x-3-5)=0}$
   $\boldsymbol{x(x-8)=0}$

   $\boldsymbol{x=0}$ ***or*** $\boldsymbol{x-8=0}$

   Solution set: $\boldsymbol{\{0, 8\}}$

   > This problem is like Lesson 17 Example 1 and Exercise 9 in Module 1.

5. $x(x-3) = 5x + 9$

$$\mathbf{x^2 - 3x = 5x + 9}$$
$$\mathbf{x^2 - 8x - 9 = 0}$$
$$\mathbf{x^2 + 1x - 9x - 9 = 0}$$
$$\mathbf{x(x+1) - 9(x+1) = 0}$$
$$\mathbf{(x+1)(x-9) = 0}$$

$\mathbf{x + 1 = 0}$ ***or*** $\mathbf{x - 9 = 0}$

Solution set: $\{\mathbf{-1, 9}\}$

To use the zero-product property, one side of the equation must equal 0, so I will rewrite the equation and collect like terms on the left side.

To use the zero-product property, I need to rewrite this expression as a product. If I split the x term creatively, I can use the distributive property repeatedly to rewrite the left side as a product.

Lesson 18: Equations Involving a Variable Expression in the Denominator

Lesson Notes

In this lesson, students understand that an equation with a variable expression in the denominator must be rewritten as a system of equations joined by an "and."

> I learned about equations joined by an "and" in Lesson 15.

Rewrite an Equation as a System of Equations

Write each equation as a system of equations excluding the values of x that lead to a denominator of 0, and then solve the equation for x.

1. $\frac{1}{4x} = x$

> I can multiply both sides by $4x$ so long as x does not equal 0.

> I can see by inspection that $\frac{1}{2}$ and $-\frac{1}{2}$ are the values that make this equation true.

> This is a system of equations, so each line of my solution should include both statements.

$$\frac{1}{4x} = x \textit{ and } x \neq 0$$
$$1 = 4x^2 \textit{ and } x \neq 0$$
$$\frac{1}{4} = x^2 \textit{ and } x \neq 0$$

$$x = \frac{1}{2} \textit{ or } x = -\frac{1}{2} \textit{ and } x \neq 0$$

Solution set: $\left\{-\frac{1}{2}, \frac{1}{2}\right\}$

2. $\frac{1}{x+3} = 9$

> The value of x that leads to a denominator of 0 is the solution to the equation $x + 3 = 0$.

$$\frac{1}{x+3} = 9 \textit{ and } x \neq -3$$
$$1 = 9(x+3) \textit{ and } x \neq -3$$
$$1 = 9x + 27 \textit{ and } x \neq -3$$
$$-26 = 9x \textit{ and } x \neq -3$$
$$-\frac{26}{9} = x \textit{ and } x \neq -3$$

Solution set: $\left\{-\frac{26}{9}\right\}$

3. $\frac{2x}{x-4} = 5x$

$$\frac{2x}{x-4} = 5x \textit{ and } x \neq 4$$

After I multiply both sides by $x - 4$, this equation can be solved using the techniques I learned in Lesson 17.

I decided to stop rewriting the "and $x \neq 4$" each time, but I cannot forget that it is still part of the process, so I will rewrite it in the last step.

$$2x = 5x(x-4)$$
$$2x = 5x^2 - 20x$$
$$0 = 5x^2 - 22x$$
$$0 = x(5x-22)$$
$$x = 0 \textit{ or } 5x - 22 = 0 \textit{ and } x \neq 4$$

$5x - 22 = 0$ when x is assigned the value $\frac{22}{5}$.

I learned to solve these types of equations in middle school, so I can add 22 and divide by 5 quickly to get the solution.

Solution set: $\left\{0, \frac{22}{5}\right\}$

4. $\frac{1}{x-3} = \frac{4}{2x+1}$

$$\frac{1}{x-3} = \frac{4}{2x+1} \textit{ and } x \neq 3 \textit{ and } x \neq -\frac{1}{2}$$

I need to multiply both sides by $x - 3$ and by $2x + 1$.

$$1(2x+1) = 4(x-3)$$
$$2x + 1 = 4x - 12$$
$$-2x + 1 = -12$$
$$-2x = -13$$
$$x = \frac{13}{2} \textit{ and } x \neq 3 \textit{ and } x \neq -\frac{1}{2}$$

Solution set: $\left\{\frac{13}{2}\right\}$

Write an Equation Given Specific Conditions

5. Write an equation that has restrictions $x \neq -2$ and $x \neq \frac{1}{2}$.

I only need to make sure I include variable expressions that lead to a dominator of 0 when x is assigned the value -2 or $\frac{1}{2}$.

It does not really matter what I put in the numerator or what I put on the other side of the equal sign.

$$\frac{1}{(x+2)\left(x - \frac{1}{2}\right)} = x + 9$$

The expression $x + 2$ is equal to 0 when x is -2. Other expressions such as $2x + 4$ or $10 + 5x$ would have worked, too.

Lesson 19: Rearranging Formulas

I need to remember that the other variable symbols are just numbers. I can solve for x just like I would if the a, c, and b were the numbers 1, 2, and 3.

Solve an Equation for x

Solve each equation for x.

1. $2a(c - 5x) = bx$

$$\mathbf{2a(c - 5x) = bx}$$
$$\mathbf{2ac - 10ax = bx}$$
$$\mathbf{2ac = 10ax + bx}$$
$$\mathbf{2ac = x(10a + b)}$$
$$\mathbf{\frac{2ac}{10a + b} = x}$$

I can apply the distributive property and then add $10ax$ to both sides to get the x terms on the same side.

I can rewrite using the distributive property to isolate x and then divide by $10a + b$.

2. $\frac{x}{y} + \frac{1}{2} = \frac{x}{z}$

$$\mathbf{\frac{x}{y} + \frac{1}{2} = -\frac{x}{z}}$$
$$\mathbf{\frac{x}{y} + \frac{x}{z} = -\frac{1}{2}}$$
$$\mathbf{x\left(\frac{1}{y} + \frac{1}{z}\right) = -\frac{1}{2}}$$
$$\mathbf{x\left(\frac{z}{yz} + \frac{y}{yz}\right) = -\frac{1}{2}}$$
$$\mathbf{x\left(\frac{z + y}{yz}\right) = -\frac{1}{2}}$$
$$\mathbf{x = -\frac{1}{2}\left(\frac{yz}{z + y}\right)}$$

I can apply the properties of equality to get the x terms on one side and the $\frac{1}{2}$ term on the other side.

If I find a common denominator, then my final answer will look neater.

Adding $\frac{1}{y} + \frac{1}{z}$ is like adding $\frac{1}{3} + \frac{1}{5}$.

$$\frac{1}{3} \cdot \frac{5}{5} + \frac{1}{5} \cdot \frac{3}{3} = \frac{5 + 3}{15}$$

Solve for a Given Variable

3. Solve for r.

$V = \pi r^2 h$

First I need to divide both sides by πh.

$$V = \pi r^2 h$$
$$\frac{V}{\pi h} = r^2$$
$$r = \sqrt{\frac{V}{\pi h}} \text{ or } r = -\sqrt{\frac{V}{\pi h}}$$

If I take the square root of both sides, then I will have two solutions because $\left(\sqrt{\frac{V}{\pi h}}\right)^2 = \frac{V}{\pi h}$. The same is true for the negative square root.

Note: The second equation will only be valid if r can be a negative number. This is a formula from geometry for calculating the volume of a cylinder, so we would not use the second formula since all the measurements must be positive quantities.

4. Solve for L.

$T = 2\pi\sqrt{\frac{L}{g}}$

I need to "undo" every operation that has been done to the variable L in order to solve for it.

$$T = 2\pi\sqrt{\frac{L}{g}}$$
$$\frac{T}{2\pi} = \sqrt{\frac{L}{g}}$$
$$\left(\frac{T}{2\pi}\right)^2 = \frac{L}{g}$$
$$g\left(\frac{T}{2\pi}\right)^2 = L$$

Squaring a square root "undoes" the operation. Then I can multiply by g.

5. Solve the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$ for v_0.

Sometimes, variable names involve subscripts. x_0 and v_0 are variable symbols, just like x or v or t.

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$x - x_0 - \frac{1}{2}at^2 = v_0 t$$
$$\frac{x - x_0 - \frac{1}{2}at^2}{t} = v_0$$
$$v_0 = \frac{x - x_0 - \frac{1}{2}at^2}{t}$$

When solving for a variable, I can rewrite the equation so the variable is on the left side.

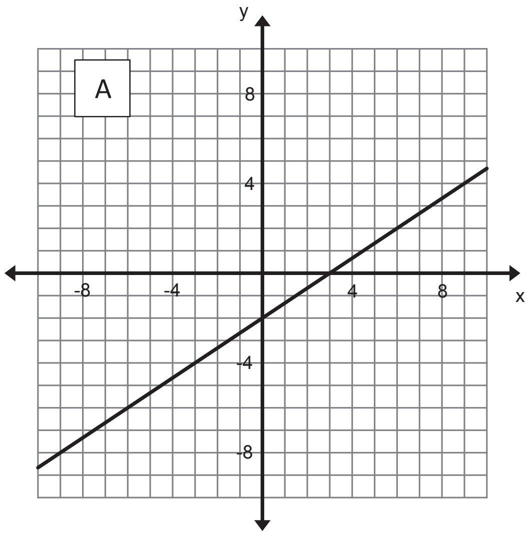
Lesson 20: Solution Sets to Equations with Two Variables

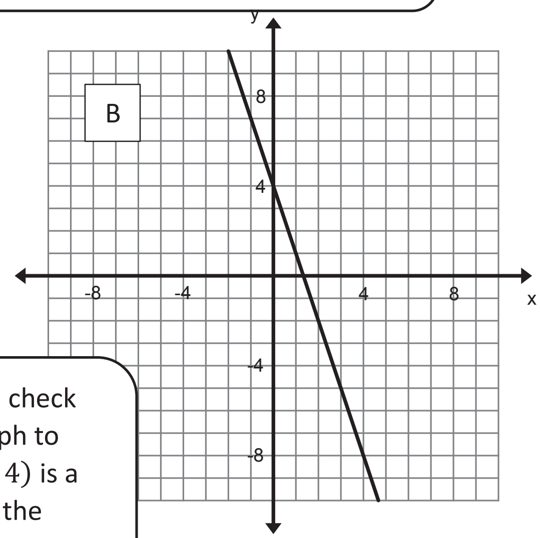
Match an Equation to the Graph of the Solution Set

Match each equation to the graph of its solution set. Explain your reasoning.

1. $3x + y = 4$
2. $4x + 3y = 12$
3. $y = \frac{2}{3}x - 2$

I can find a solution to each equation by picking an x-value and then solving to find the corresponding y-value. Then I can see if this solution is a point on the graph of the line.





I need to check each graph to see if $(0, 4)$ is a point on the graph.

Let $x = 0$. Substitute into each equation.

In Equation 1, $3(0) + y = 4$ gives a y-value of 4.

One solution is $(0, 4)$.

In Equation 2, $4(0) + 3y = 12$ gives a y-value of 4.

One solution is $(0, 4)$.

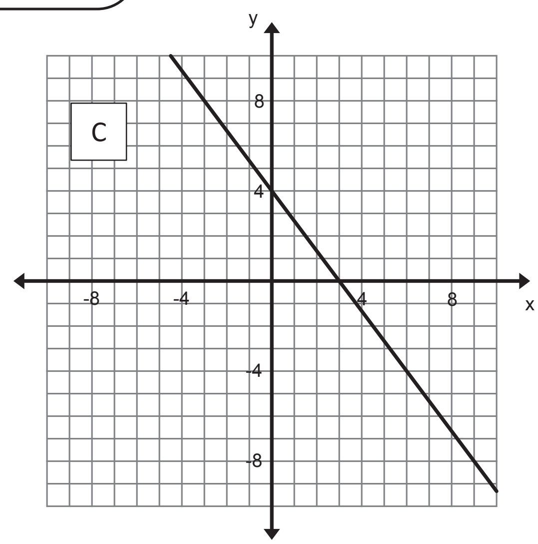
In Equation 3, $y = \frac{2}{3}(0) - 2$ gives a y-value of -2.

One solution is $(0, -2)$.

Since $(0, 4)$ is a solution to two equations, let $x = 1$.

In Equation 1, $3(1) + y = 4$, gives a y-value of 1.

One solution is $(1, 1)$.



Comparing the solutions to the graphs, Equation 1 is Graph B, Equation 2 is Graph C, and Equation 3 is Graph A.

Graph the Solution Set

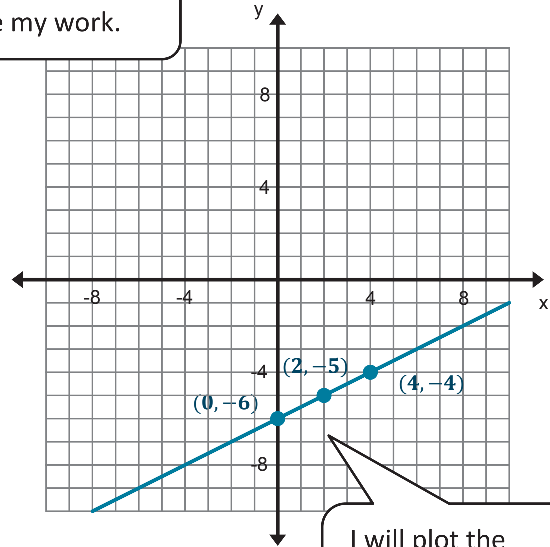
Find at least two solutions to the equation, and use them to graph the solution set. Label at least two ordered pairs that are solutions on your graph.

4. $y = \frac{x}{2} - 6$

A table will help me organize my work.

x	Substitute to Find y	Solution
$\mathbf{0}$	$\frac{0}{2} - 6 = -6$	$(0, -6)$
$\mathbf{2}$	$\frac{2}{2} - 6 = -5$	$(2, -5)$
$\mathbf{4}$	$\frac{4}{2} - 6 = -4$	$(4, -4)$

I can pick any x-values I want. For this equation, if I pick an even number, then the calculations are easier.

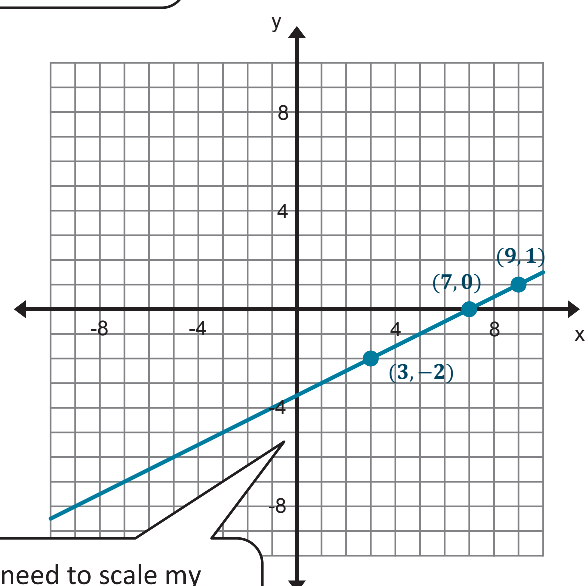


I will plot the solutions first and then draw the line.

5. $2x - 4y = 14$

I can pick either x- or y-values and then solve for the other variable.

y	Substitute to Find x	Solution
$\mathbf{0}$	$2x - 4(0) = 14$ $2x = 14$ $x = 7$	$(7, 0)$
$\mathbf{-2}$	$2x - 4(-2) = 14$ $2x = 6$ $x = 3$	$(3, -2)$
$\mathbf{1}$	$2x - 4(1) = 14$ $2x = 18$ $x = 9$	$(9, 1)$



I need to scale my graph to include all of my solutions.

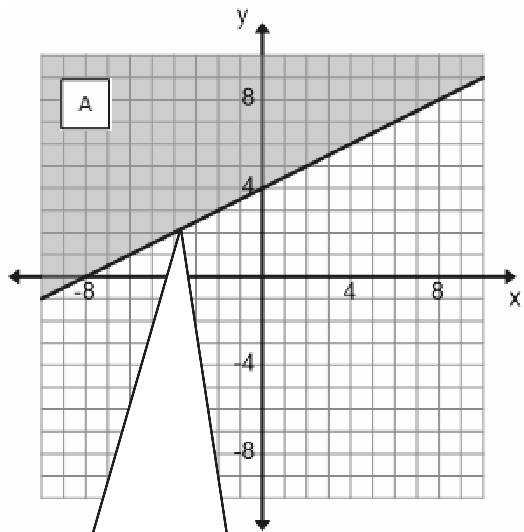
Lesson 21: Solution Sets to Inequalities with Two Variables

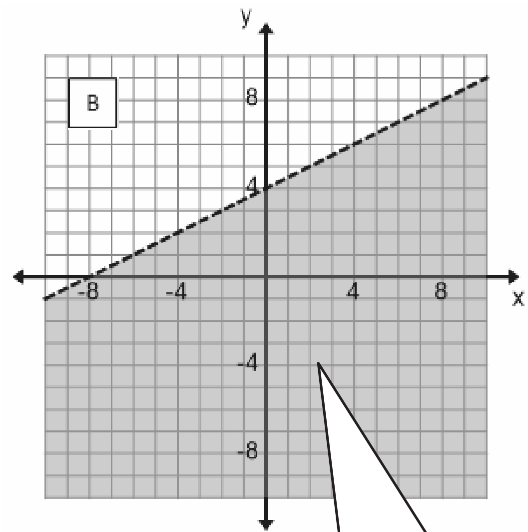
Match an Inequality to the Graph of the Solution Set

Match each inequality to the graph of its solution set. Explain your reasoning.

1. $-x + 2y < 8$
2. $y \geq \frac{1}{2}x + 4$

It is easier to compare and graph inequalities if they are solved for one variable, usually y.





The line that divides the coordinate plane into two half-planes is the same in Graph A and Graph B. This is confirmed when I solve the first inequality for y.

The shaded portion shows the solution set. When the line is dashed, the points on the line are not included in the solution set.

The first inequality when solved for y is $y < \frac{1}{2}x + 4$.

$$-x + 2y < 8$$
$$2y < x + 8$$
$$y < \frac{1}{2}x + 4$$

Add x and divide by 2 on both sides.

I can substitute any ordered pair into each inequality to see if it is a solution. If it is a solution, the half-plane that contains this point is the solution set.

The first inequality is Graph B, and the second inequality is Graph A. The ordered pairs that satisfy Inequality 1 all lie below the graph of the line $y = \frac{1}{2}x + 4$. The ordered pairs that satisfy Inequality 2 all lie above the graph of the line $y = \frac{1}{2}x + 4$.

Graph the Solution Set

To represent the solution set of an inequality, graph the related equation, and then shade the appropriate half-plane as indicated by the inequality.

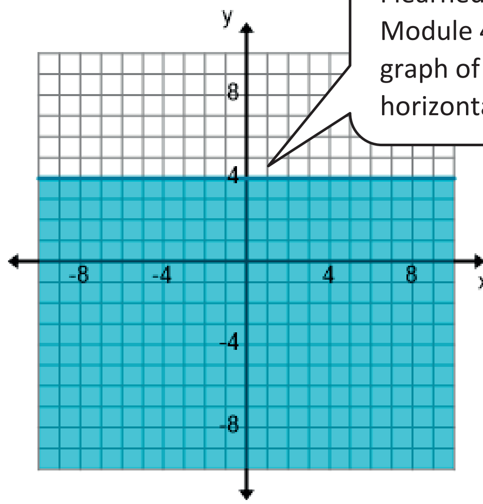
Graph the solution set in the coordinate plane. Support your answers by selecting two ordered pairs in the solution set and verifying that they make the inequality true.

3. $y \leq 4$

Related equation: $\boldsymbol{y = 4}$

Pick $\boldsymbol{(0, 1)}$***:*** $\boldsymbol{1 \leq 4}$ ***is TRUE.***

Pick $\boldsymbol{(5, -2)}$***:*** $\boldsymbol{-2 \leq 4}$ ***is TRUE.***



I learned in Grade 8 Module 4 that the graph of $y = 4$ is a horizontal line.

Since $y \leq 4$, I need to shade the half-plane that is below the line. The line $y = 4$ is included in the solution set.

4. $2x - y < 7$

$\boldsymbol{2x - y < 7}$

$\boldsymbol{2x < y + 7}$

$\boldsymbol{2x - 7 < y}$

$\boldsymbol{y > 2x - 7}$

I learned how to solve inequalities in Lesson 14 and how to rearrange formulas in Lesson 19.

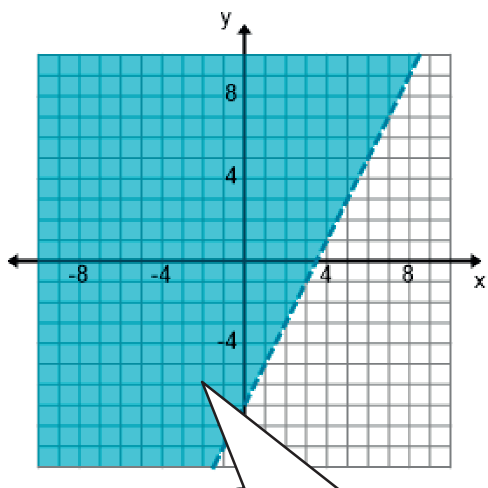
Related equation, solved for y: $\boldsymbol{y = 2x - 7}$

Pick $\boldsymbol{(0, 0)}$***:***

$\boldsymbol{2(0) - 0 < 7}$ ***is TRUE.***

Pick $\boldsymbol{(1, 1)}$***:***

$\boldsymbol{2(1) - 1 < 7}$ ***is TRUE.***



Since $y > 2x - 7$, I need to shade the half-plane that is above the line. The points on the line are not in the solution set, so I use a dashed line.

Lesson 22: Solution Sets to Simultaneous Equations

Lesson Notes

The solution set to a system of linear equations can be a single point or a line. The solution set to a system of linear inequalities is the union of two or more half-planes, represented by the portion of the coordinate plane where the graphs of the solution sets overlap.

I will identify the slope and y-intercept of each line and use them to make the graphs. I learned this in Grade 8 Module 4.

Systems of Linear Equations

1. Solve the following system of equations first by graphing and then algebraically.

$$\begin{cases} 2x - y = 6 \\ y = 3x - 2 \end{cases}$$

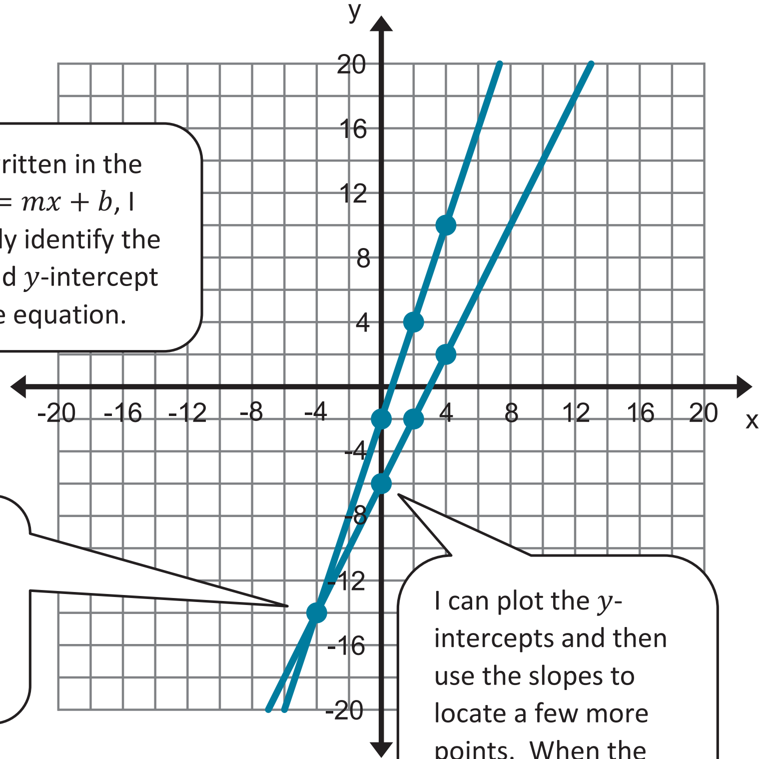
Solve the first equation for y.

$$-y = -2x + 6$$
$$y = 2x - 6$$

***Slope:* 2**

***y-intercept:* -6**

When written in the form $y = mx + b$, I can easily identify the slope and y-intercept from the equation.



The second equation has

***Slope:* 3**

***y-intercept:* -2**

***Solution:* $(-4, -14)$**

The intersection point is the solution to the system.

I can plot the y-intercepts and then use the slopes to locate a few more points. When the slope is 2, I can locate another point by counting up 2 units and right 1 unit or up 4 units and right 2 units or any other equivalent ratio.

Algebraically:

Substitute $3x - 2$ into the first equation for y, and solve for x.

$$2x - (3x - 2) = 6$$
$$2x - 3x + 2 = 6$$
$$-x = 4$$
$$x = -4$$

I can find the exact solution using algebra. A graph can only give me an estimated solution.

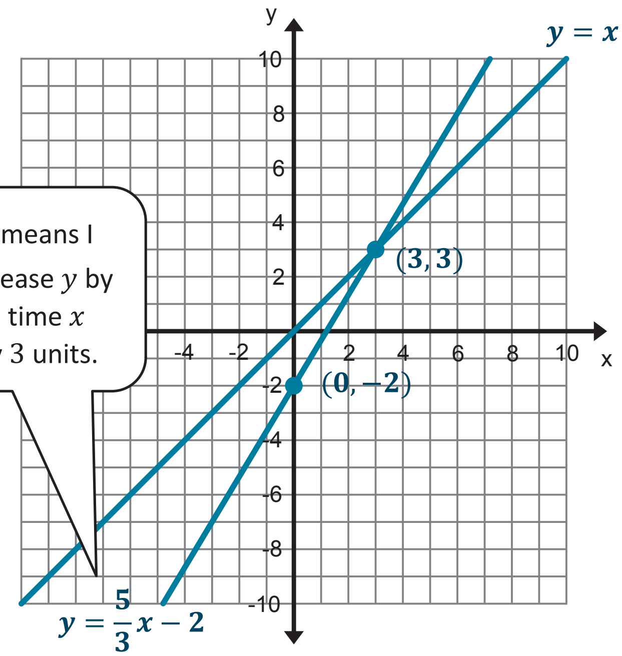
When $x = -4$, $y = 3(-4) - 2 = -14$. The solution is $(-4, -14)$.

2. Construct a system of two linear equations where $(0, -2)$ is a solution to one equation, and $(3, 3)$ is a solution to the system. Then, graph the system to show your system satisfies the given conditions.

The slope of the first equation is $\frac{3-(-2)}{3-0} = \frac{5}{3}$***, and the*** y***-intercept is*** -2***. The equation is*** $y = \frac{5}{3}x - 2$***.***
The second equation can be any equation that contains the point $(3, 3)$***. The easiest equation that has*** $(3, 3)$ ***in the solution set is*** $y = x$***.***

My second equation can be any one that has the ordered pair $(3, 3)$ in the solution set.

A slope of $\frac{5}{3}$ means I need to increase y by 5 units each time x increases by 3 units.



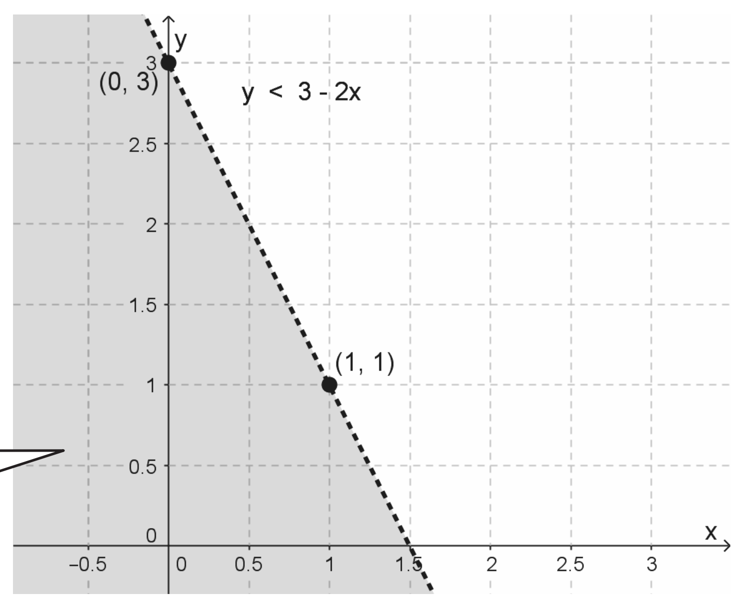
Systems of Linear Inequalities

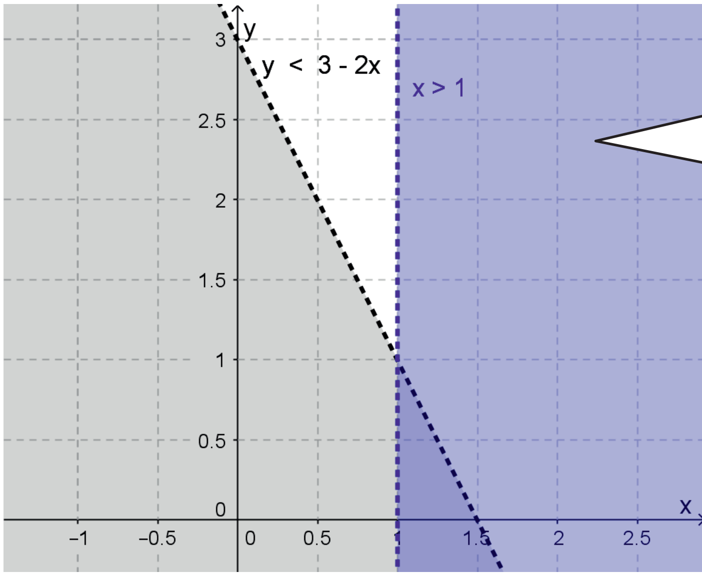
3. Graph the solution to the following system of inequalities.

$$\begin{cases} y < 3 - 2x \\ x > 1 \\ y \geq 0 \end{cases}$$

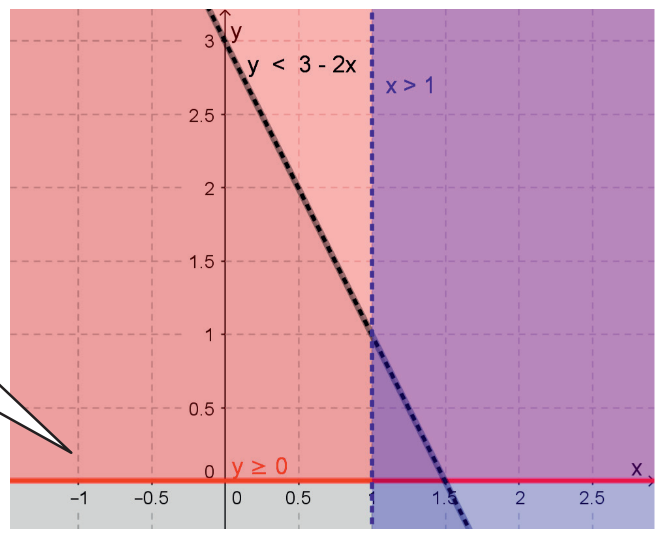
I need to graph each inequality on the same coordinate plane. The solution is where the shaded regions overlap.

I learned how to graph an inequality in Lesson 21.

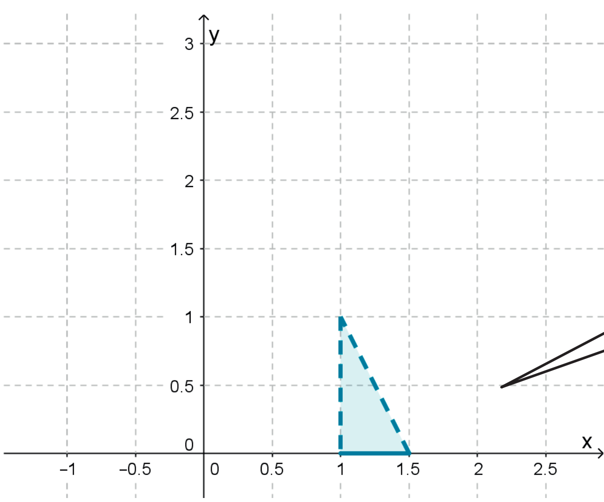




Now I can add the inequality $x > 1$. This is the half of the coordinate plane to the right of graph of the vertical line $x = 1$.



Next, I can add the inequality $y \geq 0$. The solution set includes the line $y = 0$.



The solution set is the triangular region where all three inequalities overlap.

Lesson 23: Solution Sets to Simultaneous Equations

Lesson Notes

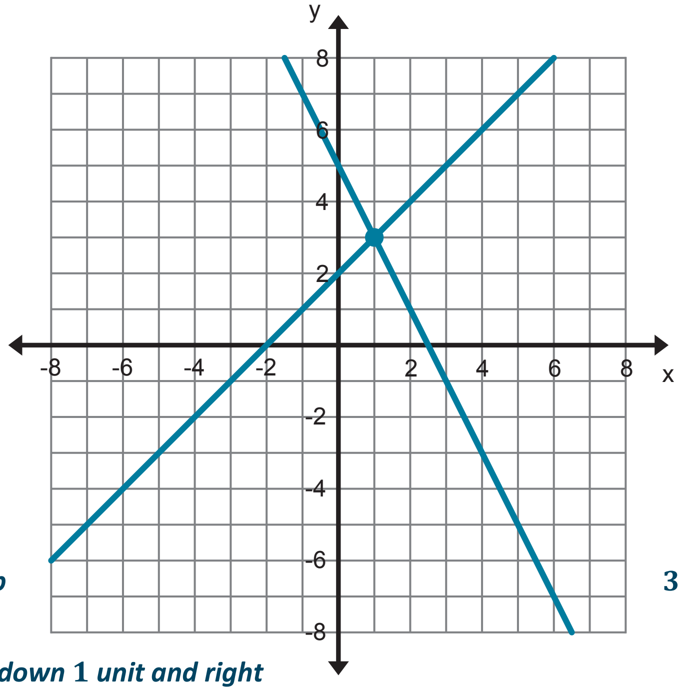
Students learned the elimination method in Grade 8. This lesson helps them understand why this method works. Students practice solving systems of linear equations using this method in the problem set.

Two Systems Can Have the Same Solution

I will use the slope and y-intercept to make my graphs.

1. Solve the system of equations $\begin{cases} y = x + 2 \\ y = -2x + 5 \end{cases}$ by graphing, and then create a new system of equations that has the same solution set. Show that both systems have the same solution.

I need to graph both equations and find the intersection point. I can check my solution by substituting into both equations.



The intersection point is $(1, 3)$.

New system of equations:

$\begin{cases} y = -x + 4 \\ y = 3x \end{cases}$

If the y-intercept is $(0, 0)$, then a slope of 3 (up 3 units and right 1 unit) will take us to $(1, 3)$.

If the y-intercept is $(0, 4)$, then a slope of -1 (down 1 unit and right 1 unit) will take us to $(1, 3)$.

Check the solution by substituting $y = 3x$ into the first equation.

$3x = -x + 4$
$4x = 4$
$x = 1$

When $x = 1$, $y = -1 + 4 = 3$. The solution is $(1, 3)$.

I need two equations whose graphs contain the point $(1, 3)$. If I pick the origin as one y-intercept and the point $(0, 4)$ as the other, I can use the graph to determine the correct slope to guarantee the lines contain $(1, 3)$.

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Solve a System Using the Elimination Method

Solve the system of equations by writing a new system that eliminates one of the variables.

2. $\begin{cases} 2x + 3y = 1 \\ 3x - 2y = 8 \end{cases}$

I need to multiply ***both*** sides of each equation by numbers that will make one of the variable terms opposites. Here I got $6y$ and $-6y$.

Multiply the first equation by $\mathbf{2}$ and the second equation by $\mathbf{3}$:

$$\mathbf{2(2x + 3y) = 2(1)}$$
$$\mathbf{3(3x - 2y) = 3(8)}$$

$$\begin{cases} \mathbf{4x + 6y = 2} \\ \mathbf{9x - 6y = 24} \end{cases}$$

When I add the two equations, one variable is eliminated. Now I can solve for the other variable.

Add the two equations together to eliminate y and solve for x: $\mathbf{13x = 26}$ ***when*** $\mathbf{x = 2}$.

Find the corresponding y-value:

$$\mathbf{2(2) + 3y = 1}$$
$$\mathbf{4 + 3y = 1}$$
$$\mathbf{3y = -3}$$
$$\mathbf{y = -1}$$

Substitute 2 for x, and solve for y.

Solution: $\mathbf{(2, -1)}$

I can check my solution by substituting it into the original equations.

Check the Solution:

Substitute $\mathbf{(2, -1)}$ into the first equation. Is this a true number sentence?

$$\mathbf{2(2) + 3(-1) = 1}$$

Yes because $\mathbf{2(2) + 3(-1) = 4 - 3 = 1}$.

Substitute $\mathbf{(2, -1)}$ into the second equation. Is this a true number sentence?

$$\mathbf{3(2) - 2(-1) = 8}$$

Yes because $\mathbf{3(2) - 2(-1) = 6 + 2 = 8}$.

The ordered pair $\mathbf{(2, -1)}$ satisfies both equations, so it is the solution to the system.

Lesson 24: Applications of Systems of Equations and Inequalities

Lesson Notes

These problems require students to create a system of equations based on a verbal description of the problem. These problems involve two related quantities. Students write two or more equations or inequalities that represent each situation and then solve the related system.

1. Jackson is a lab assistant in chemistry class. He has a solution that is 25% acid. He needs to dilute it so the final solution is 10% acid. How much water and how much 25% acid solution should he mix to achieve 32 ounces of 10% solution?

Let x represent the ounces of 25% acid solution.

Let y represent the ounces of water.

> The two quantities need to add up to 32 ounces.

$$\begin{cases} x + y = 32 \\ 0.75x + 1y = 0.90(32) \end{cases}$$

> I need to define my variables. The two quantities in this problem are ounces of water and ounces of 25% solution.

> Since I am adding pure water, I need to write this equation in terms of the concentration of water.
> 25% acid is 75% water.
> 10% acid is 90% water.

Solve the second equation for y.

$$y = -0.75x + 28.8$$

Substitute into the first equation, and solve for x.

$$\begin{aligned} x - 0.75x + 28.8 &= 32 \\ 0.25x &= 3.2 \\ x &= 12.8 \end{aligned}$$

When $x = 12.8$,

$12.8 + y = 32$. So, $y = 32 - 12.8 = 19.2$.

The solution to the system is $(12.8, 19.2)$.

Jackson needs to mix 12.8 ounces of 25% acid solution with 19.2 ounces of water.

> I need to remember what each variable represents in this problem.

2. Jane makes necklaces and earrings. It takes her 20 minutes to make a necklace and 15 minutes to make a pair of earrings. She only has enough supplies to make 60 total pieces of jewelry and has up to 18 hours available to work.

 a. What are the variables?

 She is making necklaces and earrings.

 Let x represent the number of necklaces.

 Let y represent the number of earrings.

 b. Write inequalities for the constraints.

 I know both x and y must be greater than 0 because you can't have negative amounts in this situation.

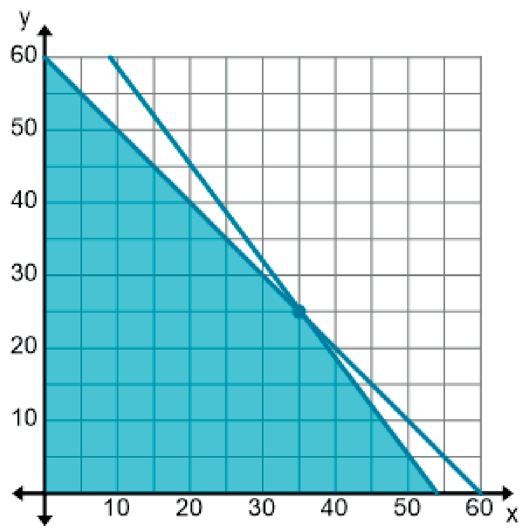
$$x \geq 0$$
$$y \geq 0$$
$$\frac{1}{3}x + \frac{1}{4}y \leq 18$$
$$x + y \leq 60$$

 I know the total number of pieces is 60, each piece takes a fraction of an hour, and the total hours are 18. For example, 20 minutes is $\frac{1}{3}$ of an hour.

 c. Graph and shade the solution set.

 I need to graph each inequality. The solution is the overlapping region. I learned this in Lesson 22.

 Since x and y are not negative, my solution set will only be in the first quadrant.



d. If she sells earrings for $\$8$ and necklaces for $\$10$, how many of each should she make to earn the most money?

She will earn the most from one of these combinations.

$\mathbf{(0, 60)}$ ***and*** $\mathbf{10(0) + 8(60) = 480}$

$\mathbf{(35, 25)}$ ***and*** $\mathbf{10(35) + 8(25) = 550}$

$\mathbf{(54, 0)}$ ***and*** $\mathbf{10(54) + 8(0) = 540}$

She should make $\mathbf{35}$ ***necklaces and*** $\mathbf{25}$ ***pairs of earrings.***

Check the points where the graphs of the lines intersect to find the optimal solution.

The point $(0,0)$ is also where two lines intersect, but that solution doesn't make sense in this situation.

e. How much will she earn with this combination of earrings and necklaces?

She will earn $\mathbf{\$550}$.

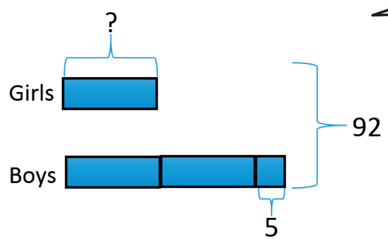
Lesson 25: Solving Problems in Two Ways—Rates and Algebra

Solving Problems Two Ways—With a Tape Diagram and with an Equation

Solve the following problem first using a tape diagram and then by setting up an equation.

1. 92 students are in an astronomy club. The number of boys in the club is 5 more than twice the number of girls. How many girls are in the astronomy club?

Method 1: Tape diagram



I need to draw two tapes, one for the number of girls and one for the number of boys.

$3 \text{ units} + 5 = 92$

$3 \text{ units} = 87$

$1 \text{ unit} = 29$

In my diagram, one unit represents the number of girls in the club.

There are 29 girls in the astronomy club.

Method 2: Equation

Number of girls: g

Number of boys: $2g + 5$

Equation: $g + (2g + 5) = 92$

$3g + 5 = 92$

$3g = 87$

$g = 29$

This equation looks like the one I came up with using the tape diagram. The tape diagram is just a visual aid to help me interpret the problem.

There are 29 girls in the astronomy club.

Check: If there are 29 girls in the astronomy club, then there must be $(2(29) + 5)$ boys, or 63 boys.

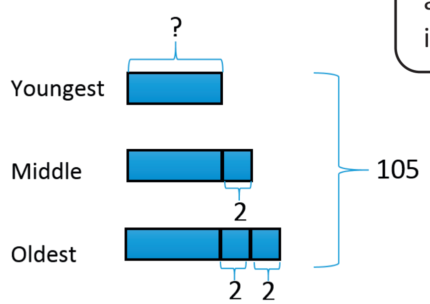
$29 + 63 = 92$ ✓

Choosing the Best Method to Solve a Problem

2. Three sisters have ages that are consecutive odd integers. If the sum of their ages is 105, what are the ages of the three sisters?

> This means their ages must increase in increments of 2.

Method 1: Tape diagram



3 units + 6 = 105

3 units = 99

1 unit = 33

> In my diagram, one unit represents the age of the youngest sister.

The youngest sister is 33 years old. The sisters' ages are 33, 35, and 37.

Method 2: Equation

> I can use my tape diagram to set up an equation. I just need to replace the "?" with a variable.

Youngest: x

Middle: $x + 2$

Oldest: $x + 4$

Equation: $x + (x + 2) + (x + 4) = 105$

$3x + 6 = 105$

$3x = 99$

$x = 33$

The sisters' ages are 33, 35, and 37.

Check: $33 + 35 + 37 = 105$ ✓

3. Martin used his credit card to purchase a souvenir while visiting another country. His credit card charged a 2.7% foreign transaction fee for the purchase. If his credit card statement showed a price of \$22.95, how much was the souvenir (without the fee)?

> I think this problem will be easier to solve using an equation than by using a tape diagram.

Let x represent the price of the souvenir.

$x(1 + 0.027) = 22.95$

$x = \frac{22.95}{1.027} \approx 22.35$

The price of the souvenir before the fee was \$22.35.

Lesson 26: Recursive Challenge Problem—The Double and Add 5 Game

Lesson Notes

In this lesson, students are introduced to the concept of a recursive sequence. This is a topic that is covered more extensively in Module 3.

Generating Terms in a Sequence when Given a Recursive Formula

1. Write down the first 5 terms of the recursive sequence defined by the initial value and recurrence relation below.

$a_1 = 5$ and $a_{i+1} = a_i + 8$, for $i \geq 1$

a_1 is the first term in the sequence (the initial value).

a_{i+1} represents the next term in the sequence after a_i. This formula is saying that to find any term in the sequence, I need to add 8 to the previous term.

I first need to interpret the notation. Since this is a recursive sequence, each term will be found by using the previous term. In order to find the terms of the sequence, I need to be given an initial value and a recurrence relation.

$\boldsymbol{a_1 = 5}$

$\boldsymbol{a_2 = a_1 + 8 = 5 + 8 = 13}$

$\boldsymbol{a_3 = a_2 + 8 = 13 + 8 = 21}$

$\boldsymbol{a_4 = a_3 + 8 = 21 + 8 = 29}$

$\boldsymbol{a_5 = a_4 + 8 = 29 + 8 = 37}$

$\boldsymbol{(5, 13, 21, 29, 37)}$

2. The following recursive sequence was generated starting with an initial value of a_0 and the recurrence relation $a_{i+1} = 8a_i$, for $i \geq 0$. Fill in the blanks of the sequence.

Sequences usually start with either term number 1 or term number 0.

(______, 48, ______, 3072 , ______, _____).

$(\mathbf{6, 48, 384, 3072, 24756, 196608})$

To find the term before 48, I need to divide by 8.

Writing Formulas in Terms of the Initial Value

3. For the recursive sequence generated by an initial value of a_0 and recurrence relation $a_{i+1} = 5a_i + 2$, for $i \geq 0$, find a formula for a_1, a_2, a_3, a_4 in terms of a_0.

$\boldsymbol{a_1 = 5a_0 + 2}$

I found this formula by replacing i with 0 in the recurrence relation.

$\boldsymbol{a_2 = 5a_1 + 2 = 5(5a_0 + 2) + 2 = 25a_0 + 12}$

This formula is in terms of a_1. Now, I can replace a_1 with the formula from above.

$\boldsymbol{a_3 = 5a_2 + 2 = 5(25a_0 + 12) + 2 = 125a_0 + 62}$

$\boldsymbol{a_4 = 5a_3 + 2 = 5(125a_0 + 62) + 2 = 625a_0 + 312}$

Lesson 27: Recursive Challenge Problem—The Double and Add 5 Game

Writing a Recursive Formula for a Sequence

For each sequence, find an initial value and recurrence relation that describes the sequence.

1. $(3, 7, 11, 15, 19, 23, 27, 31, 35, \dots)$

The initial value is 3. I will call this first term a_1.

I see that this sequence has a "plus 4" pattern. To find any term in the sequence, 4 is added to the previous term. I can use this pattern to write the recurrence relation.

$\boldsymbol{a_1 = 3 \text{ and } a_{i+1} = a_i + 4 \text{ for } i \geq 1}$

I put this inequality to indicate that I started with a_1.

2. $(1, 5, 25, 125, 625, 3125, \dots)$

I see that this sequence has a "times 5" pattern. To find any term in the sequence, the previous term is multiplied by 5.

$\boldsymbol{a_1 = 1 \text{ and } a_{i+1} = 5a_i \text{ for } i \geq 1}$

A Formula for the n^{th} Term of a Sequence

3. Show that the n^{th} term of the sequence $(1, 5, 25, 125, 625, 3125, \dots)$ could be described using the formula, $a_n = 5^{n-1}$ for $n \geq 1$.

I could rewrite this sequence as powers of 5.
$(5^0, 5^1, 5^2, 5^3, 5^4, 5^5, \dots)$
The n^{th} term of the sequence is 5^{n-1}.

$\boldsymbol{a_1 = 5^{1-1} = 5^0 = 1}$

$\boldsymbol{a_2 = 5^{2-1} = 5^1 = 5}$

$\boldsymbol{a_3 = 5^{3-1} = 5^2 = 25}$

$\boldsymbol{a_4 = 5^{4-1} = 5^3 = 125}$

Lesson 28: Federal Income Tax

Lesson Notes

In this lesson, students explore a real-world application of piecewise linear functions, federal income tax rates. Students were introduced to piecewise linear functions in Lesson 1 of this module.

Using Tax Tables and Formulas

Use the formula and tax tables provided in this lesson to perform all computations.

1. Find the taxable income of a married couple with one child who have a combined income of $\$47{,}000$.

   taxable income $=$ ***income*** $-$ ***exemption deduction*** $-$ ***standard deduction***

   taxable income $= \mathbf{\$47,000 - \$11,700 - \$12,200 = \$23,100}$

   I can find the exemption deduction and standard deduction from the tax tables given in the lesson.

2. Find the federal income tax of a married couple with one child who have a combined income of $\$47{,}000$.

   I need to use their taxable income, which I found in the first example to be \$23,100.

Federal Income Tax for Married Filing Jointly for Tax Year 2013

If Taxable Income is Over	But Not Over--	The Tax Is	Plus the Marginal Rate	Of the Amount Over
$\$0$	$\$17{,}850$	10%		$\$0$
$\$17{,}850$	$\$72{,}500$	$\$1{,}785.00$	15%	$\$17{,}850$

Since their taxable income is between these two values, I need to use this row of the table.

Federal income tax $= \mathbf{\$1,785 + 0.15(\$23,100 - \$17,850) = \$2,572.50}$

3. Find the effective federal income tax rate of a married couple with one child who have a combined income of $\$47{,}000$.

Effective federal income tax rate $= \dfrac{\textbf{federal income tax}}{\textbf{total income}} \cdot \mathbf{100\%} = \dfrac{\mathbf{\$2{,}572.50}}{\mathbf{\$47{,}000}} \cdot \mathbf{100\%} \approx \mathbf{5.5\%}$

I recall from class that the effective federal income tax rate is the actual percentage of the couple's income that was paid in taxes.