Lesson 3: Solving Problems by Finding Equivalent Ratios

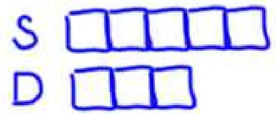
In this lesson, we learned how to solve two new types of equivalent ratio problems. First, we learned how to use a tape diagram to solve a problem where the TOTAL for the equivalent ratio is given.

Sammy and David were selling water bottles to raise money for new football uniforms. Sammy sold 5 water bottles for every 3 water bottles David sold. Together they sold 160 water bottles. How many did each boy sell?

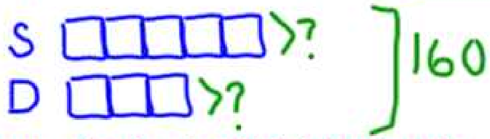
STEP 1: Read the problem.
STEP 2: STOP when you read the ratio and write it down!

5:3 - For every five water bottles Sammy sold, David sold three.

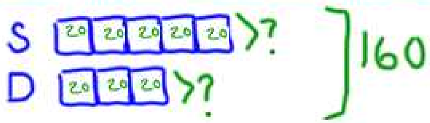
STEP 3: Draw a tape diagram.



STEP 4: Label your tape diagram with ? marks for where the answer will go and quantities given in the problem. ***(Is the quantity given just for one of the ratio quantities, the total quantity, or the difference between the two quantities?)***

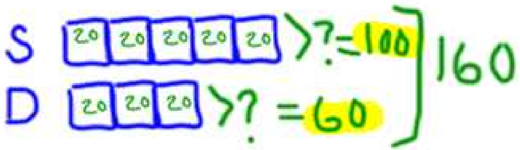


STEP 5: Divide to find the value of each unit and label all of the units.



$160 \div 8 = 20$

STEP 6: Add or multiply to find the answer to the question.



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Lesson 3: Solving Problems by Finding Equivalent Ratios

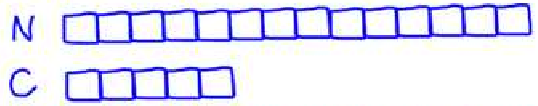
Second, we learned how to use a tape diagram to solve a problem where the DIFFERENCE BETWEEN for the equivalent ratio is given.

The Superintendent of Highways is further interested in the numbers of commercial vehicles that frequently use the county's highways. He obtains information from the Department of Motor Vehicles for the month of September and finds that for every 14 non-commercial vehicles, there were 5 commercial vehicles. If there were 108 more non-commercial vehicles than commercial vehicles, how many of each type of vehicle frequently use the county's highways during the month of September?

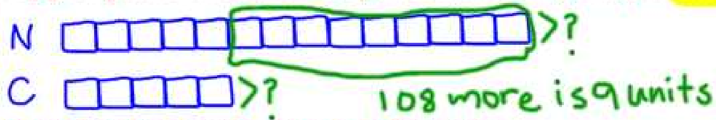
STEP 1: Read the problem.
STEP 2: STOP when you read the ratio and write it down!

14: 5 - For every 14 non-commercial vehicles there are 5 commercial vehicles.

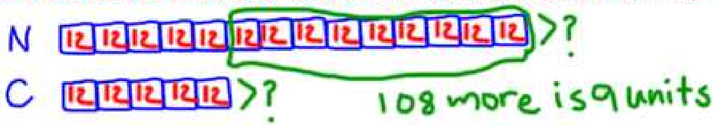
STEP 3: Draw a tape diagram.



STEP 4: Label your tape diagram with ? marks for where the answer will go and quantities given in the problem. ***(Is the quantity given just for one of the ratio quantities, the total quantity, or the difference between the two quantities?)***

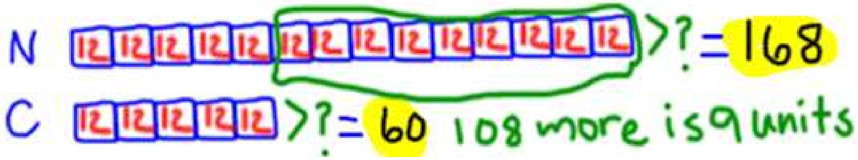


STEP 5: Divide to find the value of each unit and label all of the units.



$108 \div 9 = 12$

STEP 6: Add or multiply to find the answer to the question.



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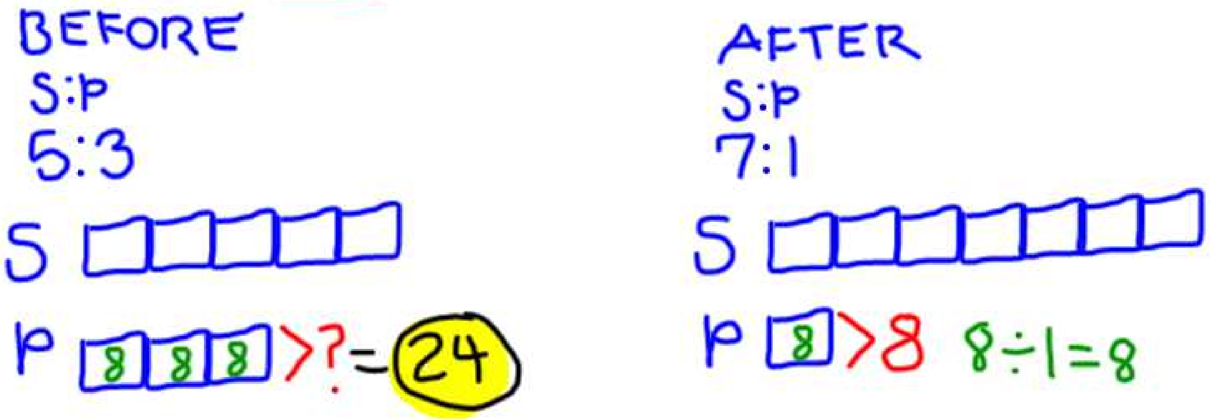
Lesson 3: Solving Problems by Finding Equivalent Ratios

Next, we learned how to use we learned how to use a BEFORE and AFTER tape diagram to solve ratio problems that give two ratios - a before ratio and after. It is important to notice that the total number in the ratios do not change, just the distribution of the ratios. This means we can use one value given for an equivalent ratio to find all of the values for the equivalent ratios.

6. Peter is trying to work out by completing sit-ups and push-ups in order to gain muscle mass. Originally, Peter was completing five sit-ups for every three push-ups, but then he injured his shoulder. After the injury, Peter completed the same amount of exercises as he did before his injury, but completed seven sit-ups for every one push-up. During a training session after his injury, Peter completed eight push-ups. How many push-ups was Peter completing before his injury?



Learning Targets

By the end of this lesson, you will be able to answer the following questions:

(1) How can tape diagrams be helpful in solving ratio word problems?
(2) How do you solve ratio problems when you are given the total of the two quantities?
(3) How do you solve ratio problems when you are given the difference between two quantities?
(4) Why is a *before* tape diagram and an *after* tape diagram useful when solving ratio word problems?

Learning Targets

Why do you need to know this?

Ratios can be used to solve all types of real world problems. We use ratios to decide what items have the best price when we shop, which cars get the best gas mileage, and many other real world problems.

Classwork

Example 1

A County Superintendent of Highways is interested in the numbers of different types of vehicles that regularly travel within his county. In the month of August, a total of 192 registrations were purchased for passenger cars and pickup trucks at the local Department of Motor Vehicles (DMV). The DMV reported that in the month of August, for every 5 passenger cars registered, there were 7 pickup trucks registered. How many of each type of vehicle were registered in the county in the month of August?

a. Using the information in the problem, write four ratios and describe the meaning of each.

The ratio given to us in the problem is 5:7. For every five cars there are seven trucks.

Another ratio that we can write is 7:5. For every seven trucks there are five cars.

Additionally, you can write the ratio 12:5. For every 12 vehicles there are 5 cars.

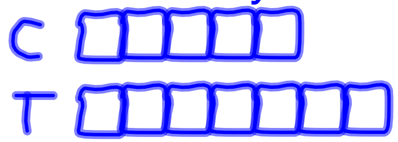
Finally, you can write the ratio 12:7. For every 12 vehicles there are 7 trucks.

There are more ratios that you can write but these are four examples.

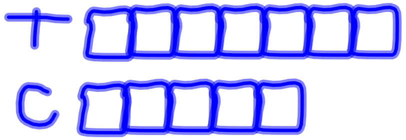
A County Superintendent of Highways is interested in the numbers of different types of vehicles that regularly travel within his county. In the month of August, a total of 192 registrations were purchased for passenger cars and pickup trucks at the local Department of Motor Vehicles (DMV). The DMV reported that in the month of August, for every 5 passenger cars registered, there were 7 pickup trucks registered. How many of each type of vehicle were registered in the county in the month of August?

b. Make a tape-diagram that represents the quantities in the ratios that you wrote.

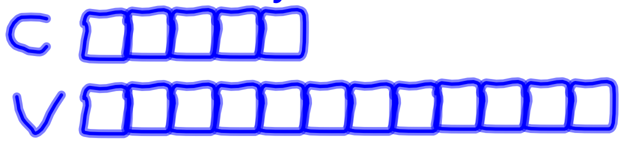
5:7 - For every five cars there are 7 trucks.



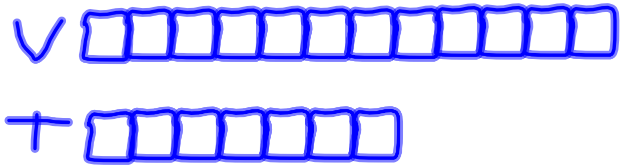
7:5 - For every seven trucks there are 5 cars.



5:12 - For every 5 cars there are 12 total vehicles.



12:7 - For every 12 total vehicles there are seven trucks.



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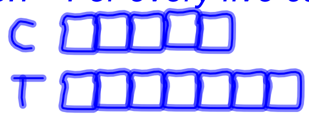
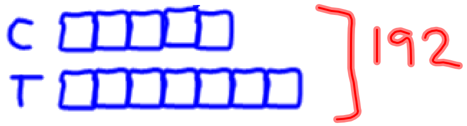
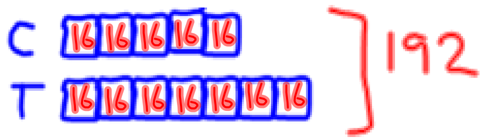


c. How many equal-sized parts does the tape diagram consist of? 12

d. What total quantity does the tape diagram represent? 192

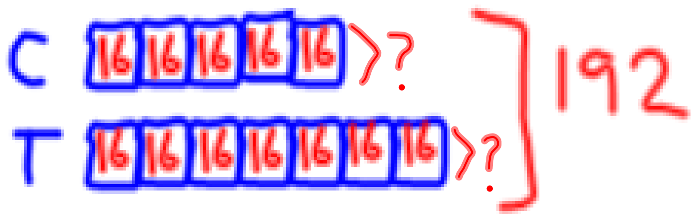


e. What value does each individual part of the tape diagram represent?



$192 \div 12 = 16$

f. How many of each type of vehicle were registered in August?



$C = 16 \times 5 = 80$

$T = 16 \times 7 = 112$

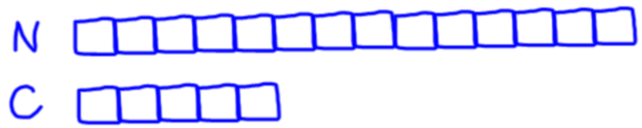
Example 2

The Superintendent of Highways is further interested in the numbers of commercial vehicles that frequently use the county's highways. He obtains information from the Department of Motor Vehicles for the month of September and finds that for every 14 non-commercial vehicles, there were 5 commercial vehicles. If there were 108 more non-commercial vehicles than commercial vehicles, how many of each type of vehicle frequently use the county's highways during the month of September?

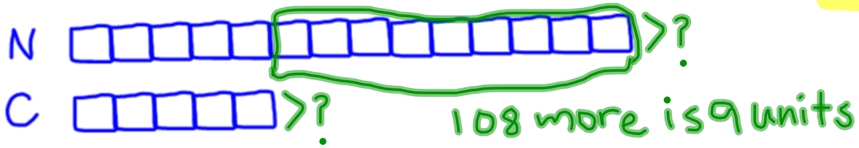
STEP 1: Read the problem.
STEP 2: STOP when you read the ratio and write it down!

14: 5 - For every 14 non-commercial vehicles there are 5 commercial vehicles.

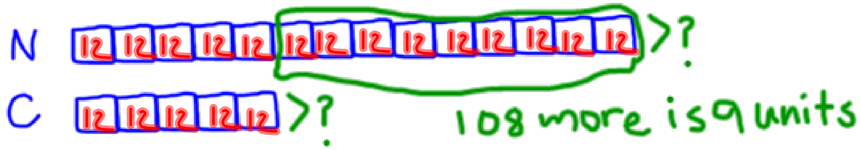
STEP 3: Draw a tape diagram.



STEP 4: Label your tape diagram with ? marks for where the answer will go and quantities given in the problem. ***(Is the quantity given just for one of the ratio quantities, the total quantity, or the difference between the two quantities?)***

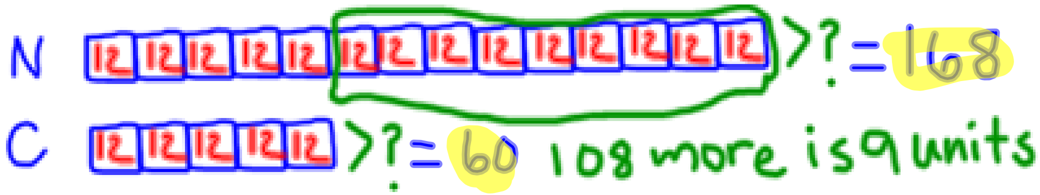


STEP 5: Divide to find the value of each unit and label all of the units.



$108 \div 9 = 12$

STEP 6: Add or multiply to find the answer to the question.



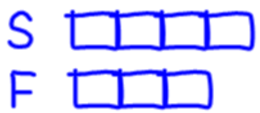
Exercises

1. The ratio of the number of people who own a smartphone to the number of people who own a flip phone is 4:3. If 500 more people own a smartphone than a flip phone, how many people own each type of phone?

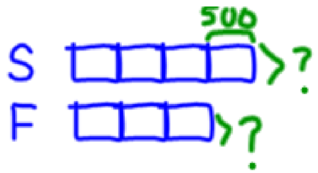
STEP 1: Read the problem.
STEP 2: STOP when you read the ratio and write it down!

4:3 - For every four people who own a smartphone, 3 people own a flip phone.

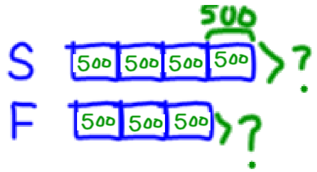
STEP 3: Draw a tape diagram.



STEP 4: Label your tape diagram with ? marks for where the answer will go and quantities given in the problem. ***(Is the quantity given just for one of the ratio quantities, the total quantity, or the difference between the two quantities?)***

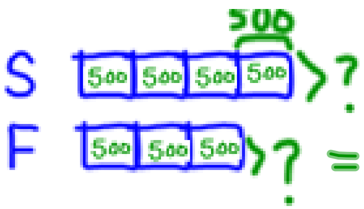


STEP 5: Divide to find the value of each unit and label all of the units.



$500 \div 1 = 500$

STEP 6: Add or multiply to find the answer to the question.



$= 500 \times 4 = 2000$

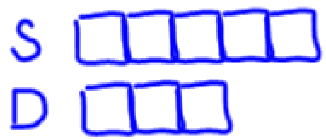
$= 500 \times 3 = 1500$

2. Sammy and David were selling water bottles to raise money for new football uniforms. Sammy sold 5 water bottles for every 3 water bottles David sold. Together they sold 160 water bottles. How many did each boy sell?

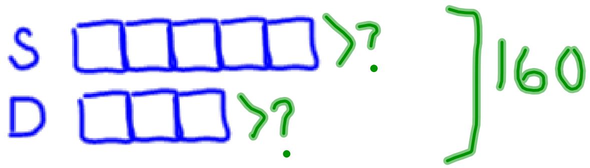
STEP 1: Read the problem.
STEP 2: STOP when you read the ratio and write it down!

5:3 - For every five water bottles Sammy sold, David sold three.

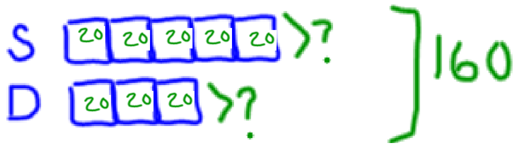
STEP 3: Draw a tape diagram.



STEP 4: Label your tape diagram with ? marks for where the answer will go and quantities given in the problem. ***(Is the quantity given just for one of the ratio quantities, the total quantity, or the difference between the two quantities?)***

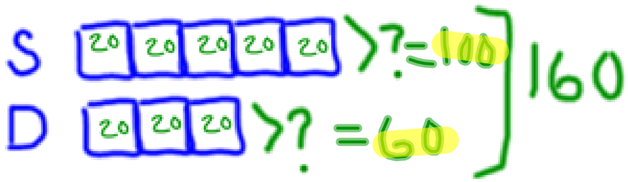


STEP 5: Divide to find the value of each unit and label all of the units.



$160 \div 8 = 20$

STEP 6: Add or multiply to find the answer to the question.

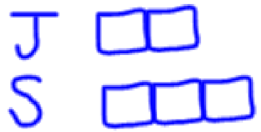


3. Ms. Johnson and Ms. Siple were folding report cards to send home to parents. The ratio of the number of report cards Ms. Johnson folded to the number of report cards Ms. Siple folded is 2:3. At the end of the day, Ms. Johnson and Ms. Siple folded a total of 300 report cards. How many did each person fold?

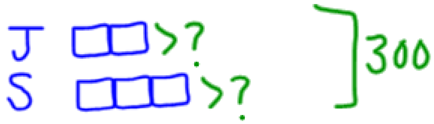
STEP 1: Read the problem.
STEP 2: STOP when you read the ratio and write it down!

2:3 - For every 2 report cards folded by Ms. Johnson, Ms. Siple folded 3.

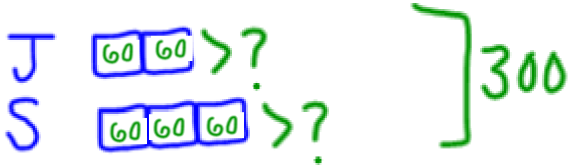
STEP 3: Draw a tape diagram.



STEP 4: Label your tape diagram with ? marks for where the answer will go and quantities given in the problem. ***(Is the quantity given just for one of the ratio quantities, the total quantity, or the difference between the two quantities?)***

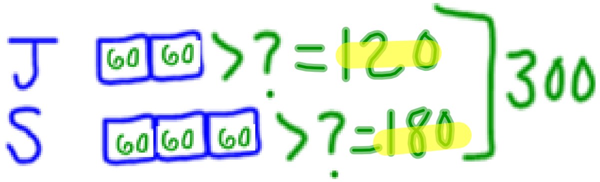


STEP 5: Divide to find the value of each unit and label all of the units.



$300 \div 5 = 60$

STEP 6: Add or multiply to find the answer to the question.



Exercises

4. At a country concert, the ratio of the number of boys to the number of girls is 2:7. If there are 250 more girls than boys, how many boys are at the concert?

B:G

2:7

B | 50 | 50 | ? = 100

250 more girls

G | 50 | 50 | 50 | 50 | 50 | 50 | 50 |

$$5\overline{)250}\ \ 50$$

Three types of ratio problems so far:

Type 1: One quantity given

The ratio of boys to girls is 3:5. If there are 15 girls, how many boys are there?

Type 2: Total of two quantities given

The ratio of boys to girls is 3:5. If there are 35 total boys and girls, how many boys and how many girls are there?

Type 3: Difference between two quantities given.

The ratio of boys to girls is 3:5. If there are 8 more girls than boys, how many boys and how many girls are there?

5. The Business Direct Hotel caters to people who travel for different types of business trips. On Saturday night there is not a lot of business travel, so the ratio of the number of occupied rooms to the number of unoccupied rooms is 2:5. However, on Sunday night the ratio of the number of occupied rooms to the number of unoccupied rooms is 6:1 due to the number of business people attending a large conference in the area. If the Business Direct Hotel has 432 occupied rooms on Sunday night, how many unoccupied rooms does it have on Saturday night?

SAT. 2:5

O

U 72 72 72 72 72 ?

SUN. 6:1

O 72 72 72 72 72 72 432

U

$$6\overline{)432}$$
$$72$$

6. Peter is trying to work out by completing sit-ups and push-ups in order to gain muscle mass. Originally, Peter was completing five sit-ups for every three push-ups, but then he injured his shoulder. After the injury, Peter completed the same amount of exercises as he did before his injury, but completed seven sit-ups for every one push-up. During a training session after his injury, Peter completed eight push-ups. How many push-ups was Peter completing before his injury?

BEFORE

S:P

5:3

S $\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}$

P $\boxed{8}\boxed{8}\boxed{8}$ > ? = 24

AFTER

S:P

7:1

S $\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}$

P $\boxed{8}$ > 8 $8 \div 1 = 8$

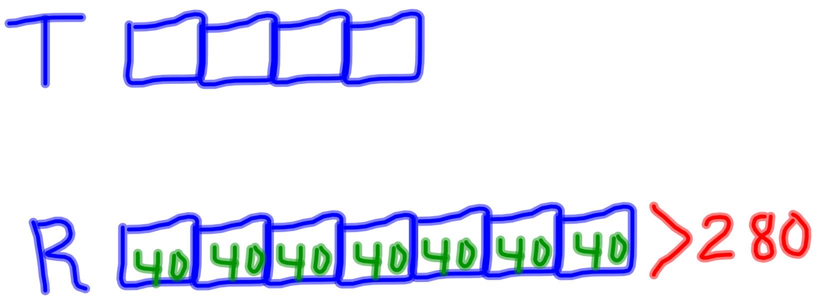
7. Tom and Rob are brothers who like to make bets about the outcomes of different contests between them. Before the last bet, the ratio of the amount of Tom's money to the amount of Rob's money was 4:7. Rob lost the latest competition, and now the ratio of the amount of Tom's money to the amount of Rob's money is 8:3. If Rob had $280 before the last competition, how much does Rob have now that he lost the bet?

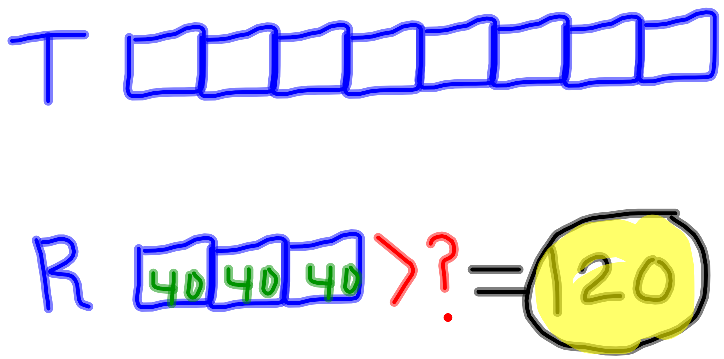
BEFORE

T:R
4:7

AFTER

T:R
8:3





$280 \div 7 = 40$

8. A sporting goods store ordered new bikes and scooters. For every 3 bikes ordered, 4 scooters were ordered. However, bikes were way more popular than scooters, so the store changed its next order. The new ratio of the number of bikes ordered to the number of scooters ordered was 5:2. If the same amount of sporting equipment was ordered in both orders and 64 scooters were ordered originally, how many bikes were ordered as part of the new order?

BEFORE

B:S
3:4

B $\boxed{}\boxed{}\boxed{}$

S $\boxed{16}\boxed{16}\boxed{16}\boxed{16}\} 64$

$64 \div 4 = 16$

AFTER

B:S
5:2

B $\boxed{16}\boxed{16}\boxed{16}\boxed{16}\boxed{16}\} ? = 80$

S $\boxed{}\boxed{}$

9. At the beginning of 6th grade, the ratio of the number of advanced math students to the number of regular math students was 3:8. However, after taking placement tests, students were moved around changing the ratio of the number of advanced math students to the number of regular math students to 4:7. How many students started in regular math and advanced math if there were 92 students in advanced math after the placement tests?

BEFORE

A:R
3:8

A | 23 | 23 | 23 | } ? = 69

R | 23 | 23 | 23 | 23 | 23 | 23 | 23 | 23 | } ? = 184

AFTER

A:R
4:7

A | 23 | 23 | 23 | 23 | } 92

R | | | | | | | |

$92 \div 4 = 23$

10. During first semester, the ratio of the number of students in art class to the number of students in gym class was 2:7. However, the art classes were really small, and the gym classes were large, so the principal changed students' classes for second semester. In second semester, the ratio of the number of students in art class to the number of students in gym class was 5:4. If 75 students were in art class second semester, how many were in art class and gym class first semester?

BEFORE

A:G
2:7

A $\boxed{15}\boxed{15}$? = 30

G $\boxed{15}\boxed{15}\boxed{15}\boxed{15}\boxed{15}\boxed{15}\boxed{15}$? = 105

AFTER

A:G
5:4

A $\boxed{15}\boxed{15}\boxed{15}\boxed{15}\boxed{15}$ 75

$75 \div 5 = 15$

G $\boxed{}\boxed{}\boxed{}\boxed{}$

11. Jeanette wants to save money, but she has not been good at it in the past. The ratio of the amount of money in Jeanette's savings account to the amount of money in her checking account was 1:6. Because Jeanette is trying to get better at saving money, she moves some money out of her checking account and into her savings account. Now, the ratio of the amount of money in her savings account to the amount of money in her checking account is 4:3. If Jeanette had $936 in her checking account before moving money, how much money does Jeanette have in each account after moving money?

BEFORE

S:C 1:6

S □

C | 156 | 156 | 156 | 156 | 156 | 156 | > 936

936 ÷ 6 = 156

AFTER

S:C 4:3

S | 156 | 156 | 156 | 156 | > ? = 624

C | 156 | 156 | 156 | > ? = 468

Learning Targets

By the end of this lesson, you will be able to answer the following questions:

(1) How can tape diagrams be helpful in solving ratio word problems?
When given a ratio and one quantity in an equivalent ratios, you can find the value of each unit in the tape diagram. This gives you all of the information you need to solve the problem.

(2) How do you solve ratio problems when you are given the total of the two quantities?
After you draw your tape diagram, you label the TOTAL of the equivalent ratio. If you divide the TOTAL number of units (boxes) into the TOTAL quantity in the equivalent ratio, you can find the answer to the question.

(3) How do you solve ratio problems when you are given the difference between two quantities?
After you draw your tape diagram, you label the DIFFERENCE between the two ratios in the equivalent ratio. If you divide the NUMBER of units that represent the difference in the tape diagram by the DIFFERENCE given in the equivalent ratio, you can find the answer to the question.

(4) Why is a *before* tape diagram and an *after* tape diagram useful when solving ratio word problems?
When you are given two different ratios in the problem that both represent the same total number, draw a tape diagram representing the ratio BEFORE and one representing the ratio AFTER. Then use the clues in the problem to find the value of each unit in the tape diagram.

Homework

Problem Set Lesson 3