


## Math & the Standards: K-12 Overview for Parents

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## Myths Versus Facts

## Arizona's College & Career Readiness Standards/Common Core Introduction Video

### How Are The Standards Different? *An Overview of the Shifts*

Arizona's College & Career Readiness Standards for Mathematics contain three key shifts to mathematics instruction.

-  1. **Focus** strongly where the standards focus\*
-  2. **Coherence:** think across grades, and link to major topics within grades
-  3. **Rigor:** in major topics\* pursue:
  - conceptual understanding,
  - procedural skill and fluency, and
  - application with equal intensity.

\*For a list of major, supporting, and additional clusters by grade, please refer to "Where to focus by grade level" at <http://achievethecore.org/shifts-mathematics>

## How Is This Different?

### *An Overview*

**Shift 1: Focus** strongly where the Standards focus

From	To
Cover content that is a “mile-wide and an inch-deep”	Assess fewer topics at each grade (as required by the Standards)
Give equal importance to all content	Dedicate large majority of score points to the major work* of the grade

## How Is This Different?

### *An Overview*

**Shift 2: Coherence:** think across grades, and link to major topics within grades

From	To
Assessment as a checklist of individual standards	Items that connect standards, clusters, and domains (as is natural in mathematics) as well as items that assess individual standards
Each topic in each year is treated as an independent event	Consistent representations are used for mathematics across the grades, and Content connects to and builds on previous knowledge

## How Is This Different?

### *An Overview*

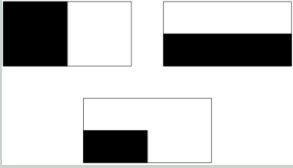
**Shift 3: Rigor:** in major topics pursue conceptual understanding, procedural skill and fluency, and application with **equal intensity**

From	To
Unbalanced emphasis on procedure or application	Assessment of all three aspects of rigor in balance
A lack of items that require conceptual understanding	Items that require students to demonstrate <b>conceptual understanding</b> of the mathematics, not just the procedures
Fluency items that are only routine and ordinary	<b>Fluency</b> items that are presented in new ways, as well as some that are routine and ordinary
Application of mathematics to routine and contrived word problems	<b>Application</b> of mathematics to authentic non-routine problems and real-world situations

SHIFT: FOCUS

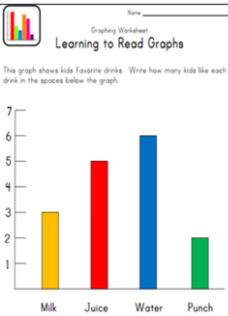
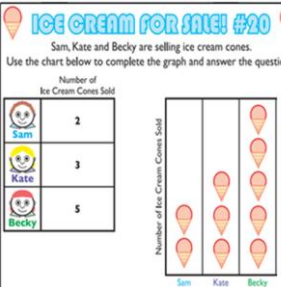
## Focus in K-8

### Grade 2 Geometry Example

Traditional Approach (Grade 2 Geometry)	AZCCRS-M-Aligned Approach (2.G.A.3)
Shawn cut a rectangle along two lines of symmetry. How many equal shares will he have?	<p>Ms. Nim gave her students a picture of a rectangle. Then she asked them to shade in one half of the rectangle. Here are three pictures:</p>  <p>Which ones show one half? Explain.</p> <p>Source: Illustrative Mathematics.  <a href="https://www.illustrativemathematics.org/illustrations/827">https://www.illustrativemathematics.org/illustrations/827</a></p>

## Focus in K-8

### Supporting Work Reinforcing Major Work

Traditional Approach (Grade 1)	AZCCRS-M-Aligned Approach (1.MD.C.4)
 <p>The graph shows kids' favorite drinks. Write how many kids like each drink in the spaces below the graph.</p> <p>Milk _____ Water _____</p> <p>Juice _____ Punch _____</p>	 <p>Sam, Kate and Becky are selling ice cream cones. Use the chart below to complete the graph and answer the questions.</p> <ol style="list-style-type: none"> <li>Who sold the most ice cream? _____</li> <li>Who sold the least ice cream? _____</li> <li>How many more ice cream cones did Becky sell than Sam? _____</li> <li>How many ice cream cones were sold in all? _____</li> </ol>

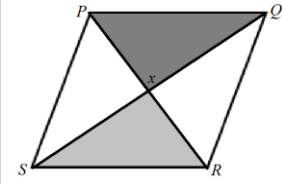
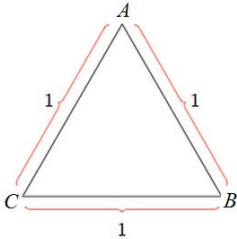
## Focus in K-8

### Supporting Work Reinforcing Major Work

Traditional Approach (Grade 7)	AZCCRS-M-Aligned Approach (7.SP.C.8)
<p>A coin is flipped three times.</p> <p><b>Part A:</b> Draw a tree diagram that shows all possible outcomes.</p> <p><b>Part B:</b> Create an organized list that shows all possible outcomes.</p>	<p>12 During an experiment, three coins were tossed once.</p> <p><b>HHH Part A:</b> Give the sample space to show all possible outcomes for tossing three coins one time, using the letter H when a coin faces "heads" up, and the letter T when it faces "tails" up.</p> <p><b>HHT</b></p> <p><b>HTH</b></p> <p><b>HTT</b></p> <p><b>THT</b></p> <p><b>TTH</b></p> <p><b>TTT</b></p> <p><b>Part B:</b> Based on your answer to part A, how many outcomes consist of 3 heads or 3 tails? <b>2</b></p> <p><b>Part C:</b> During a math class, each of 24 students tossed three coins once. Based on your answer to part B, how many students would you expect to get a result of 3 heads or 3 tails? <b><math>\frac{2}{8} = \frac{x}{24}</math></b></p> <p>Show your work.</p> <p><b><math>x = 6</math> students expected to get 3 heads or 3 tails</b></p> <p>Source: EngageNY.  <a href="http://www.engageny.org/sites/default/files/resource/attachments/math-grade-7.pdf">http://www.engageny.org/sites/default/files/resource/attachments/math-grade-7.pdf</a></p>

## Focus in High School

### Widely Applicable Prerequisites

Traditional Approach (Geometry)	AZCCRS-M-Aligned Approach (G-CO.C.10, G-SRT.C)
<p>In the diagram below of rhombus <math>PQRS</math>, decide whether there is enough information to prove that the two triangles are congruent.</p> 	<p>Below is an equilateral triangle whose side lengths are each 1 unit:</p>  <p>Find the area of <math>\triangle ABC</math>.</p> <p>Source: Illustrative Mathematics.  <a href="https://www.illustrativemathematics.org/illustrations/1322">https://www.illustrativemathematics.org/illustrations/1322</a></p>

## Focus in High School

The mile-wide inch-deep problem looks different in high school. In earlier grades it's a matter of having too many topics. In high school it's a matter of having too many separately memorized techniques, with no overall understanding of the structure to tie them altogether. So narrowing and deepening the curriculum is not so much a matter of eliminating topics, as seeing the structure that ties them together.

–Prof. William McCallum

<http://commoncoretools.me/2012/02/16/the-structure-is-the-standards/#comments>

SHIFT: COHERENCE

## Coherence Across Grades

### *Seeing the Structure*

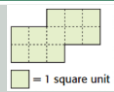
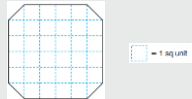
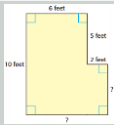

*“The Standards were not so much assembled out of topics as woven out of progressions.”*

#### What It Means

- Each standard fits into a progression with previous and future grades.

## Coherence Across Grades

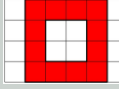
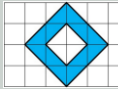
### *Consistent Progressions*

Traditional Progressions (Perimeter and Area)		
<b>Grade 3:</b>	Write the area of the shape.	
<b>Grade 4:</b>	Determine the area of the shape in square units.	
<b>Grade 5:</b>	Find the perimeter of the figure.	
<b>Grade 6:</b>	Select the rectangle with an area of 24 square units and a perimeter of 20 units.	



## Coherence Across Grades

### Consistent Progressions

AZCCRS-M-Aligned Progressions (Area and Surface Area)	
<b>3.MD.C.6:</b>	Find the area of each colored figure.  
<b>4.MD.A.3:</b>	Karl's rectangular vegetable garden is 20 feet by 45 feet, and Makenna's is 25 feet by 40 feet. Whose garden is larger in area? How much larger is that garden?
<b>5.NF.B.4b:</b>	An aerial photo of farmland shows the dimensions of a field in fractions of a mile. Create a model to show the area, in square miles, of a field that is $\frac{3}{4}$ mile by $\frac{1}{3}$ mile.
<b>6.G.A, 6.RP.A.3:</b>	Alexis needs to paint the four exterior walls of a large rectangular barn. The length of the barn is 80 feet, the width is 50 feet, and the height is 30 feet. The paint costs \$28 per gallon, and each gallon covers 420 square feet. How much will it cost Alexis to paint the barn? Explain your work.

## Coherence Across Grades

### Consistent Representations

AZCCRS-M-Aligned Items Showing Coherent Representations	
<p><b>3.OA.B.</b> Understand properties of multiplication and the relationship between multiplication and division.</p> <ol style="list-style-type: none"> <li>Apply properties of operations as strategies to multiply and divide.</li> <li>Understand division as an unknown-factor problem.</li> </ol> <p><b>Item:</b> Rewrite <math>48 \div \blacktriangle = 6</math> two ways.</p>	<p><b>6.NS.A.</b> Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</p> <ol style="list-style-type: none"> <li>Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.</li> </ol> <p><b>Item:</b> Rewrite <math>\frac{3}{8} \div \frac{3}{4} = x</math> two ways. Explain what dividing by <math>\frac{3}{4}</math> means.</p>

Source: Student Achievement Partners. For illustrative purposes only.

## Coherence Within a Single Grade

### AZCCRS-M-Aligned Item at the Cluster Level (5.NBT.A: Understand the place value system)

#### *Are these equivalent to 9.52?*

Isaiah is thinking of the number 9.52 in his head. Decide whether each of these has the same value as 9.52 and discuss your reasoning.

- a. Nine and fifty-two tenths
- b.  $9 + 0.5 + 0.02$
- c. 9 ones + 5 tenths + 2 hundredths
- d.  $(9 \times 1) + \left(5 \times \frac{1}{10}\right) + \left(2 \times \frac{1}{100}\right)$
- e. 952 tenths
- f. 952 hundredths

Source: Illustrative Mathematics. <https://www.illustrativemathematics.org/illustrations/1813>

SHIFT: RIGOR

## Rigor

AZCCRS-M-aligned materials will be balanced by distributing across the three aspects of rigor:



## Rigor

### *Conceptual Understanding*

Traditional Approach to Conceptual Understanding (Grade 2)	AZCCRS-M-Aligned Approach to Conceptual Understanding (2.NBT.A)
<p>Source: Achieve the Core. <a href="http://achievethecore.org/page/253/thinking-about-place-value-in-grade-2">http://achievethecore.org/page/253/thinking-about-place-value-in-grade-2</a></p>	<p>6) <math>106 = \underline{1} \text{ hundred} + \underline{\quad} \text{ tens} + \underline{\quad} \text{ ones}</math></p> <p>7) <math>106 = \underline{\quad} \text{ tens} + \underline{\quad} \text{ ones}</math></p> <p>8) <math>106 = \underline{\quad} \text{ ones}</math></p> <p>9) <math>90 + 300 + 4 = \underline{\quad}</math></p> <p>Are these comparisons true or false?</p> <p>10) <math>2 \text{ hundreds} + 3 \text{ ones} &gt; 5 \text{ tens} + 9 \text{ ones}</math></p> <p>11) <math>0 \text{ tens} + 2 \text{ hundreds} + 4 \text{ ones} &lt; 024</math></p>

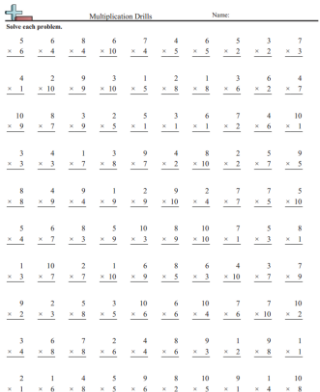
## Rigor

### Conceptual Understanding

Traditional Approach to Conceptual Understanding (Grade 6)	AZCCRS-M-Aligned Approach to Conceptual Understanding (6.EE.A)
<p>Factor:</p> $6y + 24$ <p>Expand:</p> $7(b + 5)$	<p>Circle all the expressions that are equivalent.</p> $7(b + 5) + 3 \qquad b + 38$ $7b + 7 \times 8 \qquad 7b + 38$ $7b + (7 \times 5) + 3$ <p>Show that the expressions you circled above are equivalent.</p> <p>Source: Achieve the Core.  <a href="http://achievethecore.org/page/910/extending-previous-understandings-of-properties-mini-assessment-detail-pg">http://achievethecore.org/page/910/extending-previous-understandings-of-properties-mini-assessment-detail-pg</a></p>

## Rigor

### Procedural Skill and Fluency

Traditional Approach to Procedural Skill and Fluency (Grade 3)	AZCCRS-M-Aligned Approach to Procedural Skill and Fluency (3.OA.C.7)										
	<table border="1"> <tr> <td><math>9 \times 2 = \underline{\quad}</math></td><td><math>\underline{\quad} \times 7 = 56</math></td></tr> <tr> <td><math>24 \div 6 = \underline{\quad}</math></td><td><math>5 \times 8 = \underline{\quad}</math></td></tr> <tr> <td><math>7 \times 6 = \underline{\quad}</math></td><td><math>27 \div 3 = \underline{\quad}</math></td></tr> <tr> <td><math>35 \div 5 = \underline{\quad}</math></td><td><math>64 \div 8 = \underline{\quad}</math></td></tr> <tr> <td><math>9 \times \underline{\quad} = 36</math></td><td><math>\underline{\quad} \times 7 = 21</math></td></tr> </table> <p>Source Achieve the Core:  <a href="http://achievethecore.org/page/861/multiplication-and-division-within-100">http://achievethecore.org/page/861/multiplication-and-division-within-100</a></p>	$9 \times 2 = \underline{\quad}$	$\underline{\quad} \times 7 = 56$	$24 \div 6 = \underline{\quad}$	$5 \times 8 = \underline{\quad}$	$7 \times 6 = \underline{\quad}$	$27 \div 3 = \underline{\quad}$	$35 \div 5 = \underline{\quad}$	$64 \div 8 = \underline{\quad}$	$9 \times \underline{\quad} = 36$	$\underline{\quad} \times 7 = 21$
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$35 \div 5 = \underline{\quad}$	$64 \div 8 = \underline{\quad}$										
$9 \times \underline{\quad} = 36$	$\underline{\quad} \times 7 = 21$										

## Rigor

### *Procedural Skill and Fluency*

Traditional Approach to Procedural Skill and Fluency (Algebra I or II)	AZCCRS-M-Aligned Approach to Procedural Skill and Fluency (A-REI.B.4)								
<p><b>Directions:</b> Solve each equation.</p> <table> <tr> <td>[1] <math>x^2 - 8x + 7 = 0</math></td><td>[7] <math>x^2 - 10x + 25 = 0</math></td></tr> <tr> <td>[2] <math>x^2 + 1x - 2 = 0</math></td><td>[8] <math>x^2 - 5x + 6 = 0</math></td></tr> <tr> <td>[3] <math>x^2 - 6x + 5 = 0</math></td><td>[9] <math>x^2 - 2x - 8 = 0</math></td></tr> <tr> <td>[4] <math>x^2 - 6x + 8 = 0</math></td><td>[10] <math>x^2 + 0x - 4 = 0</math></td></tr> </table>	[1] $x^2 - 8x + 7 = 0$	[7] $x^2 - 10x + 25 = 0$	[2] $x^2 + 1x - 2 = 0$	[8] $x^2 - 5x + 6 = 0$	[3] $x^2 - 6x + 5 = 0$	[9] $x^2 - 2x - 8 = 0$	[4] $x^2 - 6x + 8 = 0$	[10] $x^2 + 0x - 4 = 0$	<p><b>Solve.</b> <math>\frac{3}{4}c(c - 1) = c</math></p> <p><b>Solve.</b> <math>(x + 2)(4x - 1) = 2x(5x -</math></p>
[1] $x^2 - 8x + 7 = 0$	[7] $x^2 - 10x + 25 = 0$								
[2] $x^2 + 1x - 2 = 0$	[8] $x^2 - 5x + 6 = 0$								
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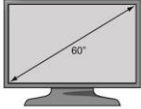
## Rigor

### *Application*

Traditional Approach to Application (Grade 2)	AZCCRS-M-Aligned Approach to Application (2.OA.A.1)
<p>Jose starts with 7 bottle caps. He gets 2 more from Rebecca. How many bottle caps does Jose end with?</p> <p>Ashley collects 47 apples. Ashley's father gives Ashley 3 more. How many apples does Ashley have?</p> <p>There are 3 erasers. 36 erasers more are added. How many are there total?</p>	<p>Jim has 11 red pens and 29 blue pens.</p> <p>How many pens does Jim have?</p> <p>Since the first day of school, 4 students have left Sam's class. Sam's class now has 25 students.</p> <p>How many students were in Sam's class on the first day of school?</p> <p>43 students are playing tag at recess. 25 students are playing soccer.</p> <p>Which <b>two</b> equations can be used to find how many fewer students are playing soccer than playing tag?</p> <p>a. <math>25 + ? = 43</math>  b. <math>? - 25 = 43</math>  c. <math>43 + 25 = ?</math>  d. <math>43 - 25 = ?</math></p> <p>Source: Achieve the Core.  <a href="http://achievethecore.org/page/258/representing-and-solving-addition-and-subtraction-problems-mini-assessment-detail-pg">http://achievethecore.org/page/258/representing-and-solving-addition-and-subtraction-problems-mini-assessment-detail-pg</a></p>

# Rigor

## *Not Always Together, Not Always Separate*

Traditional Approach Ignoring Combined Rigor (Algebra I)	AZCCRS-M-Aligned Approach to Combining Aspects of Rigor (A-REI.B.4, A-CED.A.1)
<p>Use the given information to write the equation of each quadratic function.</p> <p>2. Its graph is a parabola with x-intercepts (2, 0) and (-1, 0) and y-intercept (0, 6).</p> <p style="text-align: right;">equation: _____</p> <p>3. The function has zeros (5, 0) and (1, 0) and <math>f(0) = 1</math>.</p> <p style="text-align: right;">equation: _____</p>	<p>5. TV screens are measured on the diagonal. The diagram below shows a 60" TV screen.</p>  <p>For this TV, the ratio of height to width<sup>2</sup> is 0.618.</p> <ol style="list-style-type: none"> <li>What is the area of this TV screen? (2 points)</li> <li>For any TV screen with a height-to-width ratio of 0.618, write a function <math>A(d)</math> that gives the screen area <math>A</math>, in square inches, when the length of the diagonal is <math>d</math> (measured in inches). (1 point)</li> <li>The cost of making any size TV screen is <math>0.0373A(d) + 0.5d + 10</math>. What is the largest screen size that can be built for \$75? (2 points)</li> </ol> <p>Source: <a href="http://achievethecore.org/page/976/quadratic-equations-mini-assessment">Achievethecore.org</a>.  <a href="http://achievethecore.org/page/976/quadratic-equations-mini-assessment">http://achievethecore.org/page/976/quadratic-equations-mini-assessment</a></p>